

Physics & diagnostics in Tokamak plasmas

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Thomson scattering

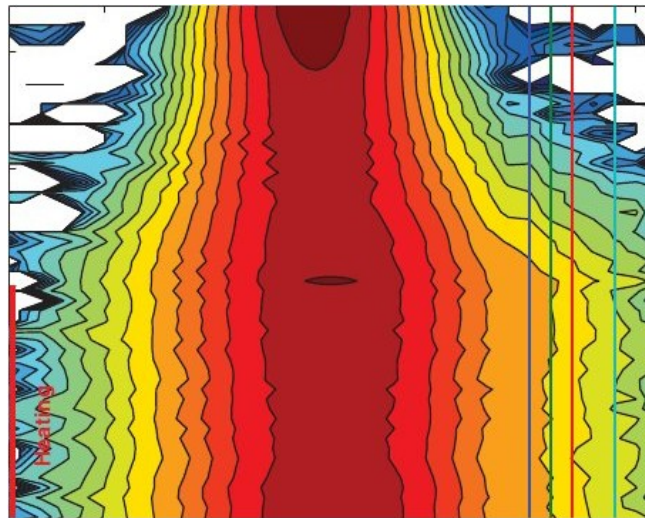
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Physics and Diagnostics in Tokamak plasmas

How to measure

- electron and ion
- density, velocity and temperature
- in the plasma core ?

Lecture

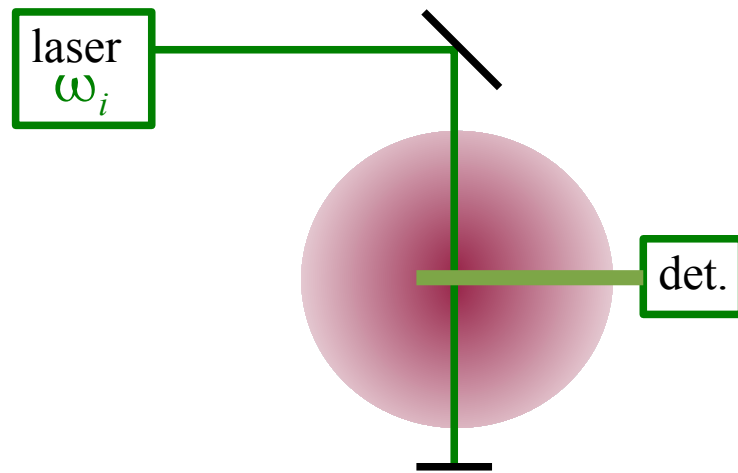
■ I. Thomson Scattering

- Thomson Scattering principle
- Incoherent Thomson Scattering
 - ITS and electron dynamics
 - Relativistic particle case
 - Magnetic field effect
 - ITS applications (T3, JET, ITER)
- Coherent Thomson Scattering
 - CTS and ion and electron dynamics
 - CTS applications (TEXTOR, ASDEX-U, ITER)

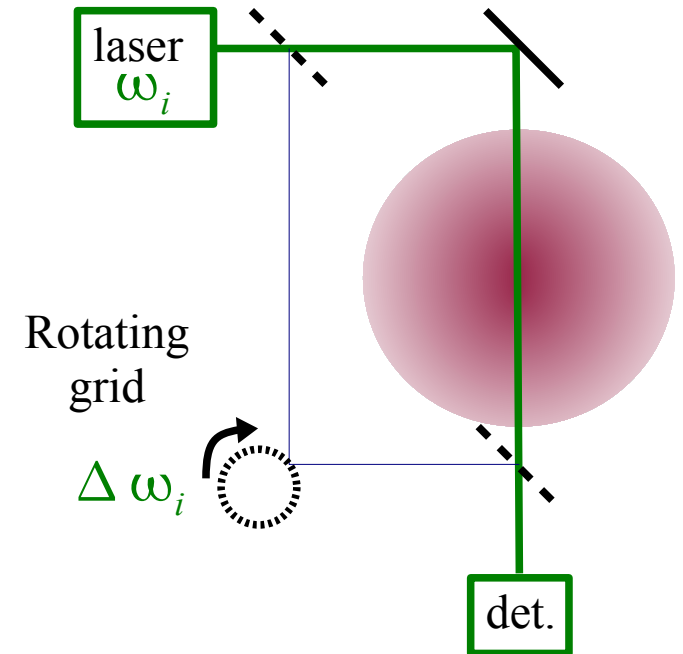
Thomson Scattering principle

Thomson Scattering principle

Scattering is different from interferometry : instead of detecting the phase shift of the initial beam through the plasma, we look at the light remitted in every other direction by the plasma.



Thomson
Scattering



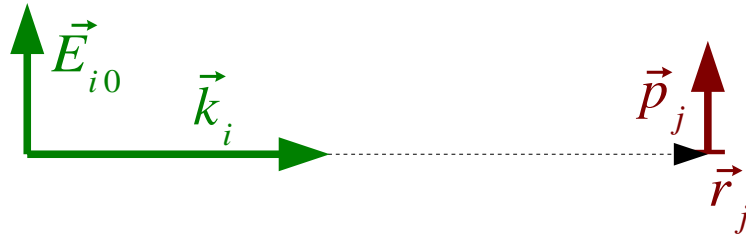
Interferometry

Scattered light intensity will give information on the **electron density**.
Doppler effect will give information on particle **velocity distributions**.

Thomson Scattering : free electron scatterer

Simple model : **free electrons** are the scatterers

the scatterers are **non relativistic**, **no magnetic field**



Mono-chromatic transverse linear mono-mode wave : $\vec{k}_i, \lambda_i, \omega_i$ $\omega_i \gg \omega_p$

$$\vec{E}_i(\vec{r}, t) = e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)} \vec{E}_{i0}$$

The incident wave electric field accelerates mainly plasma **free electrons**

$\vec{r}_j(t)$: single electron position

$$m_e \vec{a}_j(t) = -q_e \vec{E}_i(\vec{r}_j(t), t) \quad q_e > 0$$

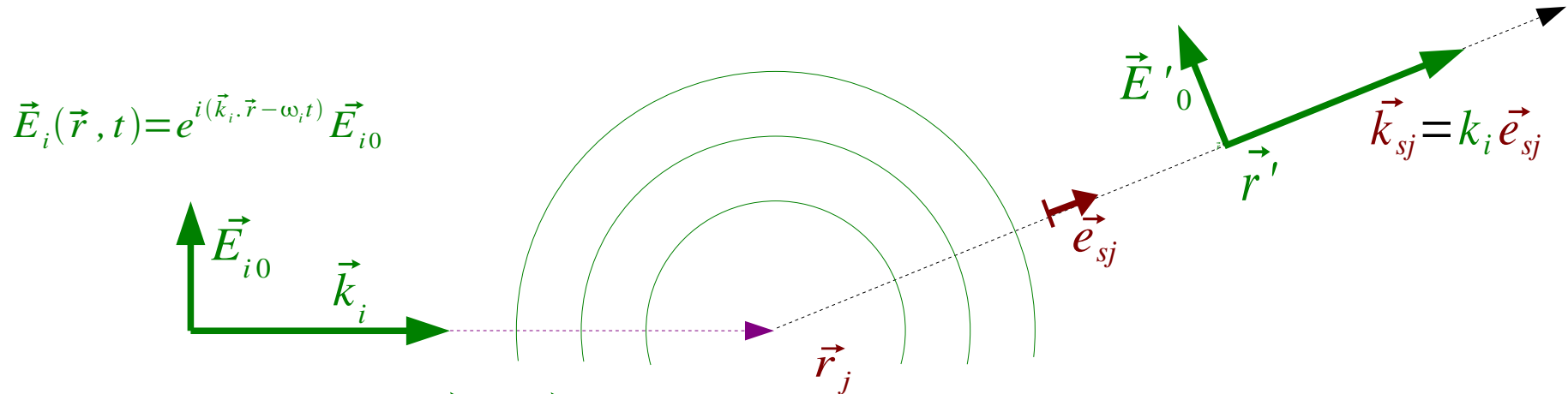
The electron position oscillates at the incident wave frequency ω_i :

$$\vec{r}_j(t) = \frac{q_e}{m_e \omega_i^2} \vec{E}_i(\vec{r}_j(t), t)$$

Each single electron oscillation creates a local oscillating dipole :

$$\vec{p}_j(t) = -q_e \vec{d}[\vec{r}_j(t), t] = \frac{-q_e^2}{m_e \omega_i^2} \vec{E}_i(\vec{r}_j(t), t)$$

Thomson Scattering : electric field scattered by a dipole



The scattered field ($\vec{E}_{sj}$, $\vec{H}_{sj}$) is the Maxwell-Ampere and Maxwell-Faraday equation solutions with a dipole as the source :

$$\vec{\nabla} \wedge \vec{H}_{sj}(\vec{r}', t) = \partial_t (\epsilon_0 \vec{E}_{sj}(\vec{r}', t) + \vec{p}_j(t) \delta(\vec{r}' - \vec{r}_j)) \quad \partial_t = \frac{\partial}{\partial t}$$

$$\vec{\nabla} \wedge \vec{E}_{sj}(\vec{r}', t) = -\mu_0 \partial_t \vec{H}_{sj}(\vec{r}', t)$$

Both equation combined in a wave equation with a punctual source :

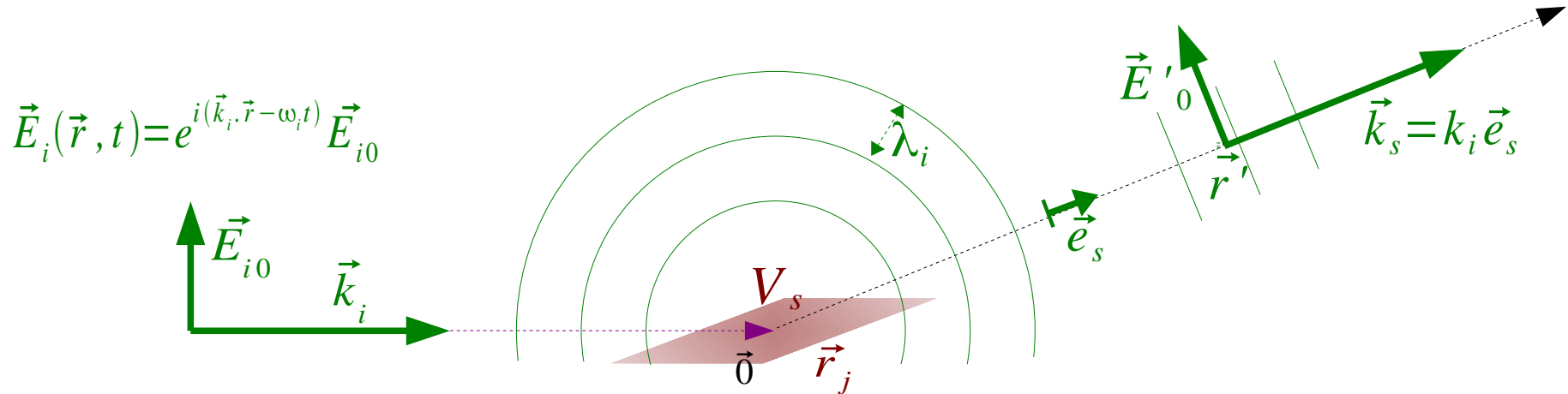
$$-\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{E}_{sj}(\vec{r}', t)) + \frac{\omega_i^2}{c^2} \vec{E}_{sj}(\vec{r}', t) = -\mu_0 \vec{p}_j(t) \delta(\vec{r}' - \vec{r}_j)$$

The wave solution is a Green function corresponding to a spherical wave :

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{ik_i|\vec{r}' - \vec{r}_j|}}{|\vec{r}' - \vec{r}_j|} e^{i(\vec{k}_i \cdot \vec{r}_j - \omega_i t)} \vec{e}_{sj} \wedge (\vec{e}_{sj} \wedge \vec{E}_{i0})$$

Thomson scattering is elastic (no energy absorption nor emission) : $|\vec{k}_{sj}| = |\vec{k}_i|$

Thomson Scattering : far field scattered electric field



Single free electron scattered field :

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{ik_i|\vec{r}' - \vec{r}_j|}}{|\vec{r}' - \vec{r}_j|} e^{i(\vec{k}_i \cdot \vec{r}_j - \omega_i t)} \vec{e}_{sj} \wedge (\vec{e}_{sj} \wedge \vec{E}_{i0})$$

Far field approximation :

$|\vec{r}'| \gg |\vec{r}_j|$: observation distance is much larger than scattering volume V_s dimension.

V_s is defined by crossing of **incident beam** and detector **antenna beam**.

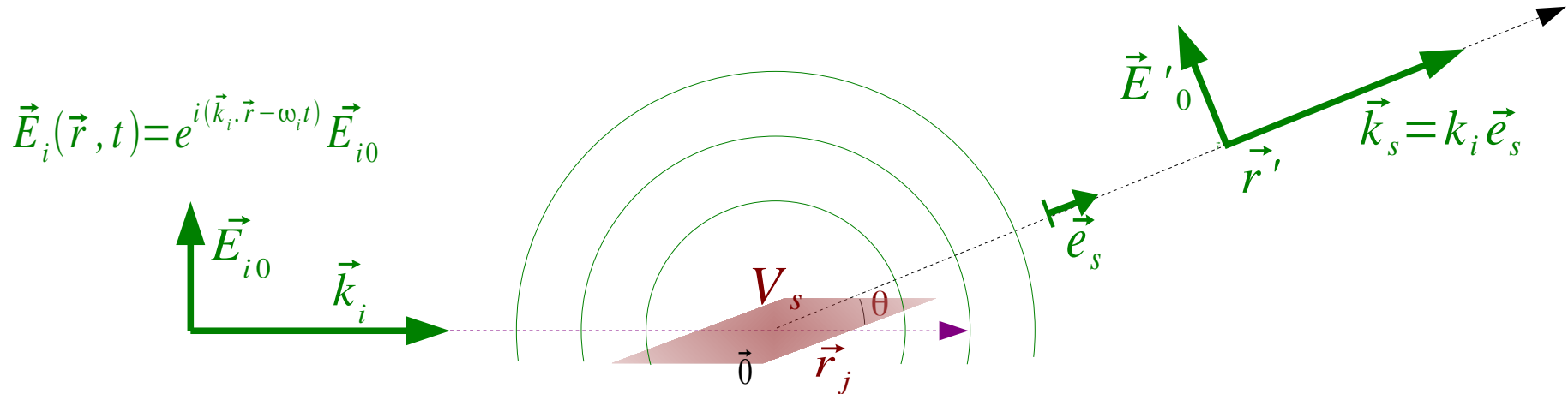
$|\vec{r}'| \gg \lambda_i$: observation distance is much larger than the incident wave wavelength.

→ One common observation direction : $\vec{e}_{sj} \sim \vec{e}_s$ $\vec{k}_s = k_i \vec{e}_s$ $\frac{1}{|\vec{r}' - \vec{r}_j|} \sim \frac{1}{r'}$

→ Spherical locally considered as plane wave : $e^{k_i|\vec{r}' - \vec{r}_j|} \sim e^{k_i \vec{e}_s \cdot (\vec{r}' - \vec{r}_j)} = e^{\vec{k}_s \cdot \vec{r}'} e^{-\vec{k}_s \cdot \vec{r}_j}$

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_j(t)}$$

Thomson Scattering : scattered electric field factors



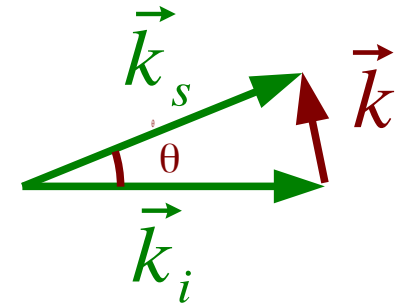
Single free electron scattered field with far field approximation :

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_j(t)}$$

$$\vec{k} = \vec{k}_s - \vec{k}_i$$

Scattering wave vector (Bragg's condition)

$$k = 2 k_i \sin(\theta/2)$$



$e^{-i(\vec{k}_s - \vec{k}_i) \cdot \vec{r}_j(t)} = e^{-i\vec{k} \cdot \vec{r}_j(t)}$: the electron specific phase function of the scattering wave vector

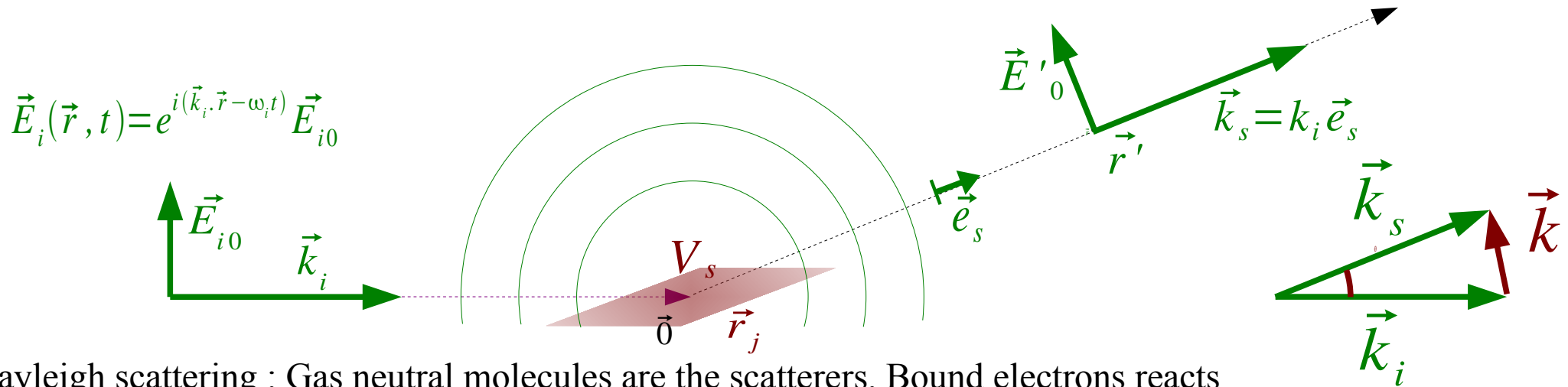
$\vec{E}'_0 = \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0})$: wave polarization modification

$e^{-i\omega_i t}$: source frequency phase

$\frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'}$: wave spherical structure

$\frac{\mu_0 q_e^2}{4\pi m_e} = r_0 = 2.8 \cdot 10^{-15} \text{ m}$: electron classical radius

Thomson Scattering vs Rayleigh Scattering



Rayleigh scattering : Gas neutral molecules are the scatterers. Bound electrons reacts to the initial wave. **Neutral molecules get polarized :**

$$\vec{p}_j(t) = \alpha_j \epsilon_0 \vec{E}_i(\vec{r}_j(t), t)$$

α_j : molecule polarizability : $\alpha_{air} = 4\pi 1.65 \cdot 10^{-30} m^3$

Far field approximation of the Rayleigh scattered electric field :

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\pi}{n_w^2 \lambda_i^2} \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) \alpha_j e^{-i\vec{k} \cdot \vec{r}_j}$$

r_0 the electron classical radius is replaced with the averaged Rayleigh radius : $r_R = \frac{\pi \langle \alpha \rangle}{n_w^2 \lambda_i^2}$

n_w : medium refraction index

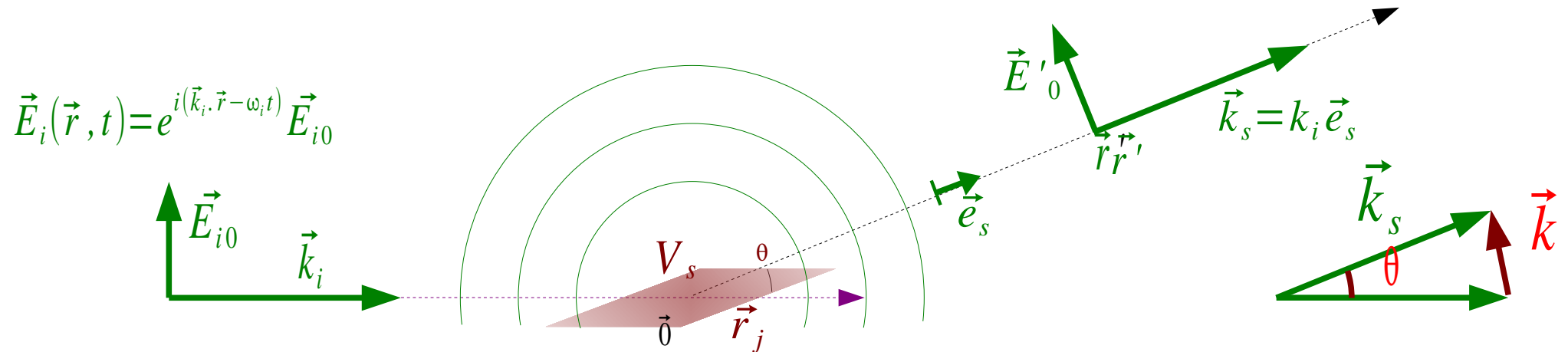
$$r_{Rair} \sim r_0 \quad \lambda_i \sim 0.15 \mu m$$

The Rayleigh radius depends on initial wave wavelength : it is larger for smaller wavelength.

The Rayleigh radius is smaller than r_0 for wavelength larger than UV wavelengths.

Rayleigh (and Raman) scattering might be used to **calibrate** Thomson scattering diagnostic.

Thomson scattering anisotropy



Far field single electron scattered field :

$$\vec{E}_{sj}(\vec{r}', t) = r_0 \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

The emission is not isotropic. It depends on the **angle** between the **wave polarization** and the **observation direction** :

$$\theta_{pol} = \pi/2 - (\vec{e}_s, \vec{E}_{i0}) \quad E'_{i0} = |\vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0})| = E_{i0} \cos \theta_{pol}$$

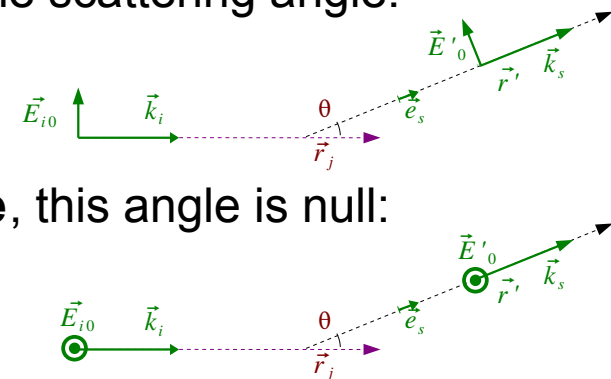
When the **wave polarization is in the scattering plane**, this angle is the scattering angle:

$$\theta_{pol} = \theta \quad E'_{i0} = E_{i0} \cos \theta$$

When the **wave polarization is perpendicular to the scattering plane**, this angle is null:

$$\theta_{pol} = 0 \quad E'_{i0} = E_{i0}$$

The scattered electric field is reduced when θ_{pol} angle goes to 90° .



Thomson scattering cross section per electron

Far field single electron scattered field :

$$\vec{E}_{sj}(\vec{r}', t) = r_0 \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

The scattered electric field amplitude :

$$|\vec{E}_{sj}(\vec{r}', t)| = r_0 \frac{1}{r'} E_{i0} \cos \theta_{pol}$$

For each **solid angle** scattered direction (θ, φ) , the electromagnetic **scattered intensity** compared to the initial wave intensity :

$$\frac{d\sigma_j}{d\Omega_s}(\theta, \varphi) = \frac{1}{I_i} \frac{dI_{sj}(\theta, \varphi)}{d\Omega_s} = \frac{r'^2 I_{sj}}{I_i} = \frac{r'^2 \langle |E_{sj}|^2 \rangle}{\langle |E_{i0}|^2 \rangle}$$

So :

$$\frac{d\sigma_j}{d\Omega_s}(\theta, \varphi) = r_0^2 \cos^2 \theta_{pol}$$

Averaged for both possible initial polarizations :

$$\frac{d\sigma_j}{d\Omega_s}(\theta, \varphi) = r_0^2 \frac{1 + \cos^2 \theta}{2}$$

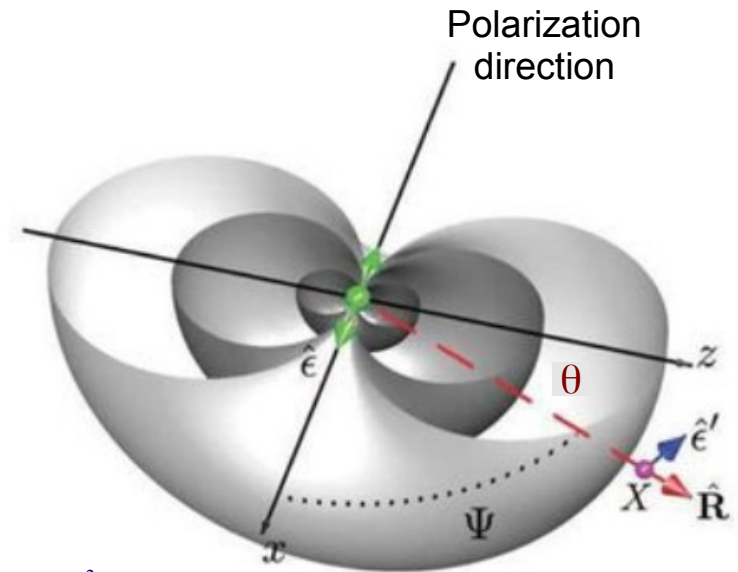
Integrated for all solid angles :

$$\sigma_{Tj} = \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2\pi} \frac{d\sigma_j}{d\Omega_s}(\theta, \varphi) \sin \theta d\varphi$$

The Thomson scattering **cross section** for a single electron :

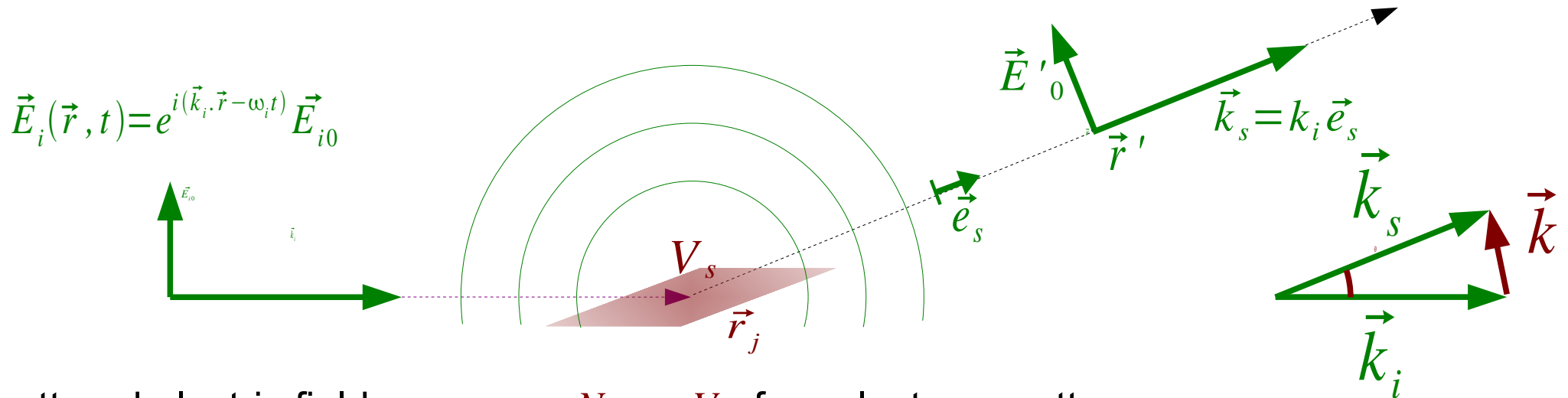
$$\sigma_{Tj} = \frac{8\pi}{3} r_0^2$$

$$r_0 = \frac{\mu_0 q_e^2}{4\pi m_e}$$



Incoherent Thomson Scattering

Thomson Scattering : scatterer sum



Scattered electric field sum upon $N_s = n_e V_s$ free electron scatterers :

$$\vec{E}_s(\vec{r}', t) = \sum_{j=1}^{N_s} \vec{E}_{sj}(\vec{r}', t) \quad \vec{E}_{sj}(\vec{r}', t) = r_0 \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} e^{-i\omega_i t} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

$$\vec{E}_s(\vec{r}', t) = r_0 \frac{e^{i\vec{k}_s \cdot \vec{r}'}}{r'} \vec{e}_s \wedge (\vec{e}_s \wedge \vec{E}_{i0}) e^{-i\omega_i t} \sum_{j=1}^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

Each scatterer scattered electric field differs by **the scattering phase**.

The **scattering signal** will be defined by the sum of the scattering phases :

$$n_{\vec{k}}(t) = \sum_{j=1}^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

Incoherent Thomson Scattering : signal correlation

$$n_{\vec{k}}(t) = \sum_{j=1}^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

Scattering signal time auto-correlation :

$$C_{\vec{k}}(\tau) = \langle n_{\vec{k}}(\vec{r}', t) n_{\vec{k}}^*(\vec{r}', t + \tau) \rangle_t = \langle \sum_{j=1}^{N_s} \sum_{l=1}^{N_s} e^{-i(\vec{k} \cdot \vec{r}_j(t) - \vec{k} \cdot \vec{r}_l(t + \tau))} \rangle_t$$

When incident wave wavelength is much shorter than the Debye length,

$$k \lambda_D > 1$$

even the collective effects due to ion shielding by electron will not affect scattering phase coherence between electrons for such a small wavelength.

For a small scattering wavelength, the cross-correlations terms are negligible :

$$C_{\vec{k}}(\tau) = \langle \sum_{j=1}^{N_s} e^{-i(\vec{k} \cdot \vec{r}_j(t) - \vec{k} \cdot \vec{r}_j(t + \tau))} \rangle_t + \langle \sum_{j \neq l}^{N_s(N_s-1)} e^{-i(\vec{k} \cdot \vec{r}_j(t) - \vec{k} \cdot \vec{r}_l(t + \tau))} \rangle_t$$

For $\tau = 0$

$$\langle |n_{\vec{k}}(t)|^2 \rangle_t = C_{\vec{k}}(0) = \langle \sum_{j=1}^{N_s} e^{i0} \rangle_t = N_s$$

This is finite power signal.

Incoherent Thomson Scattering : correlation and velocity

Scattering signal time correlation only includes electron self scattering :

$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{-i(\vec{k} \cdot \vec{r}_j(t) - \vec{k} \cdot \vec{r}_j(t+\tau))} \right\rangle_t \quad n_{\vec{k}}(t) = \sum_j^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

We assume that for τ delays shorter than $C_{\vec{k}}(\tau)$ correlation time, electron has **constant velocity** trajectory : $\vec{r}_j(t) - \vec{r}_j(t+\tau) = \vec{v}_j(t) \tau$

The velocity still varies at larger time scales.

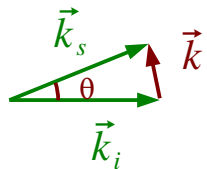
$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{i\vec{k} \cdot \vec{v}_j(t) \tau} \right\rangle_t$$

Using **ergodic** hypothesis, we introduce electron velocity distribution : $f_{e\vec{v}}$

$$C_{\vec{k}}(\tau) = N_s \iiint d\vec{v} f_{e\vec{v}}(\vec{v}) e^{i\vec{k} \cdot \vec{v} \tau}$$

The velocity component along $\vec{k}$ only plays a role : $v_k = \vec{v} \cdot \vec{e}_k$

$$\vec{e}_k = \frac{1}{k} \vec{k}$$



f_{ev_k} : probability distribution for electron velocity component along $\vec{k}$

$$C_{\vec{k}}(\tau) = N_s \int dv_k f_{ev_k}(v_k) e^{ik v_k \tau}$$

Incoherent Thomson Scattering and velocity distribution

$$C_{\vec{k}}(\tau) = N_s \int d v_k f_{e v_k}(v_k) e^{i \vec{k} \cdot \vec{v}_k \tau}$$

$$n_{\vec{k}}(t) = \sum_j^{N_s} e^{-i \vec{k} \cdot \vec{r}_j(t)}$$

The frequency auto-spectrum for a finite power signal :

$$n_{\vec{k}T}(\omega) = \int_t^{t+T} dt n_{\vec{k}}(t) e^{i \omega t}$$

$$N_{\vec{k}}(\omega) = \lim_{T \rightarrow +\infty} \frac{1}{T} |n_{\vec{k}T}(\omega)|^2$$

The frequency auto-spectrum is the time Fourier transform of the auto-correlation :

$$N_{\vec{k}}(\omega) = \int d \tau C_{\vec{k}}(\tau) e^{i \omega \tau}$$

$$N_{\vec{k}}(\omega) = N_s \int d \tau \int d v_k f_{e v_k}(v_k) e^{i(\omega + \vec{k} \cdot \vec{v}_k) \tau}$$

Integration variable substitution : $\vec{k} \tau \rightarrow 1/u$

$$N_{\vec{k}}(\omega) = N_s \frac{2\pi}{k} \int d v_k f_{e v_k}(v_k) \delta\left(\frac{\omega}{k} + v_k\right)$$

$$N_{\vec{k}}(\omega) = N_s \frac{2\pi}{k} f_{e v_k}\left(\frac{-\omega}{k}\right)$$

The Thomson scattering signal spectrum reproduces
the 1D electron velocity distribution along $\vec{k}$.

Knowing the normalization, it is proportional to the **plasma density**.

Incoherent Thomson Scattering : form factor

$$\langle |n_{\vec{k}}(t)|^2 \rangle_t = N_s$$

$$n_{\vec{k}}(t) = \sum_j^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

Scalar Form Factor for Incoherent Scattering :

$$S_{\vec{k}} = \frac{1}{N_s} \langle |n_{\vec{k}}(t)|^2 \rangle_t = 1$$

Dynamical Form Factor for Incoherent Scattering :

$$S_{\vec{k}}(\omega) = \frac{1}{N_s} N_{\vec{k}}(\omega)$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{ev_k}\left(\frac{-\omega}{k}\right)$$

Dynamical Form Factor for Incoherent Scattering is normalized :

$$\int S_{\vec{k}}(\omega) d\omega = 1$$

$$\int f_{ev_k}(v_k) dv_k = 1$$

Incoherent Thomson Scattering : scattered power

The scattered power for a scattering volume V_s is the sum of the averaged scattered power for one electron multiplied by the electron number in the volume : $N_s = n_e V_s$

$$\frac{d\sigma}{d\Omega_s}(\theta, \varphi) = \frac{r'^2 I_s}{I_i} = \frac{r'^2 \langle \left| \sum_{j=1}^{N_s} E_{sj} \right|^2 \rangle}{\langle |E_{i0}|^2 \rangle} = \frac{r'^2 \sum_{j=1}^{N_s} \langle |E_{sj}|^2 \rangle}{\langle |E_{i0}|^2 \rangle} \quad \text{Scattering cross section per solid angle}$$

$$\frac{d\sigma}{d\Omega_s}(\theta, \varphi) = N_s r_0^2 \cos^2 \theta_{pol} \quad \frac{r'^2 \langle |E_{sj}|^2 \rangle}{\langle |E_{i0}|^2 \rangle} = r_0^2 \cos^2 \theta_{pol}$$

The plasma scattered power per unit volume integrated for all directions :

$$\sigma_T = \frac{8\pi}{3} r_0^2 N_s$$

Scattered light cross section frequency spectrum

$$\frac{d\sigma}{d\Omega_s}(\theta, \varphi, \omega) = N_s r_0^2 \cos^2 \theta_{pol} \frac{2\pi}{k} f_{ev_k} \left(\frac{-\omega}{k} \right)$$

Incoherent Thomson Scattering : Maxwellian distribution

When the velocity distribution is Maxwellian,

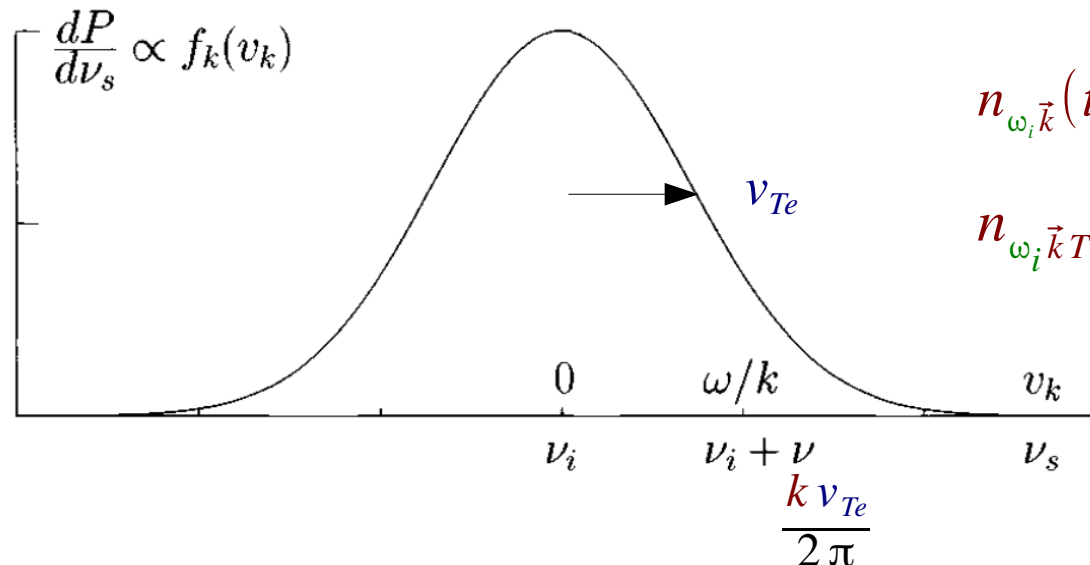
$$f_{v_k e 0}(v_k) = \frac{1}{v_{Te} \sqrt{2\pi}} e^{-\frac{v_k^2}{2v_{Te}^2}} \quad v_{Te} = \sqrt{\frac{k_B T_e}{m_e}}$$

we can deduce **the electron temperature** from the Gaussian width.

$$S_{\vec{k}}(\omega) = \frac{\sqrt{2\pi}}{k v_{Te}} e^{-\frac{\omega^2}{2k^2 v_{Te}^2}} \quad \Delta \omega_{e-1/2} = k v_{Te}$$

We can also measure the **electron density** :

$$\frac{d\sigma}{d\Omega_s}(\theta, \varphi) = n_e V_s r_0^2 \cos^2 \theta_{pol}$$



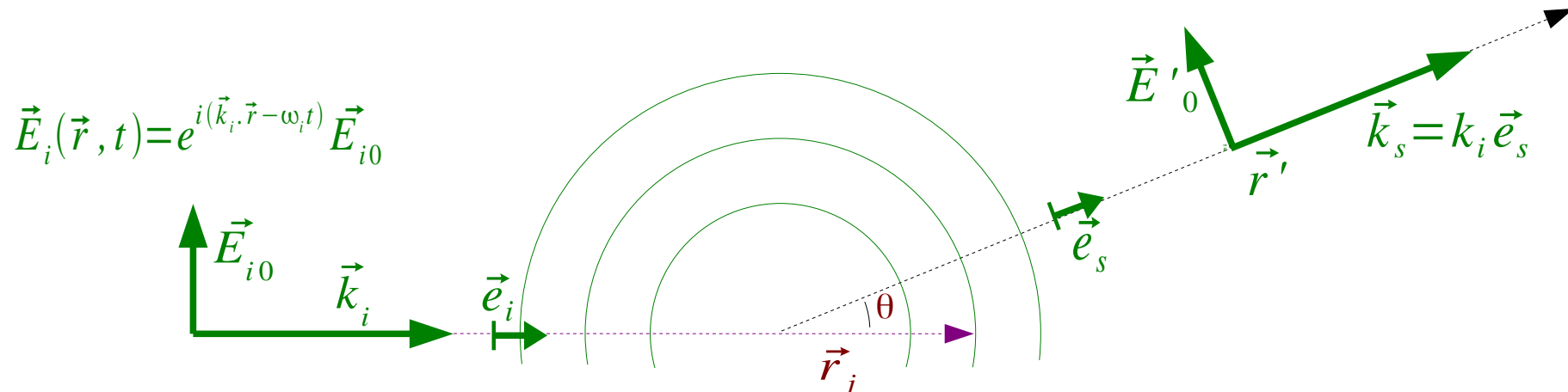
$$n_{\omega_i \vec{k}}(t) = e^{-i\omega_i t} \sum_{j=1}^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

$$n_{\omega_i \vec{k} T}(\omega) = n_{\vec{k} T}(\omega - \omega_i)$$

$$v_k = \lambda (v - v_i)$$

$$v_i = \omega_i / 2\pi$$

Thomson Scattering : relativistic acceleration



For electron with kinetic energy larger than 1 *keV*, relativistic effects have to be taken into account.

$$\vec{\beta}_j(t) = \frac{1}{C} \vec{v}_j(t)$$

Classical electron acceleration by the incident electric field :

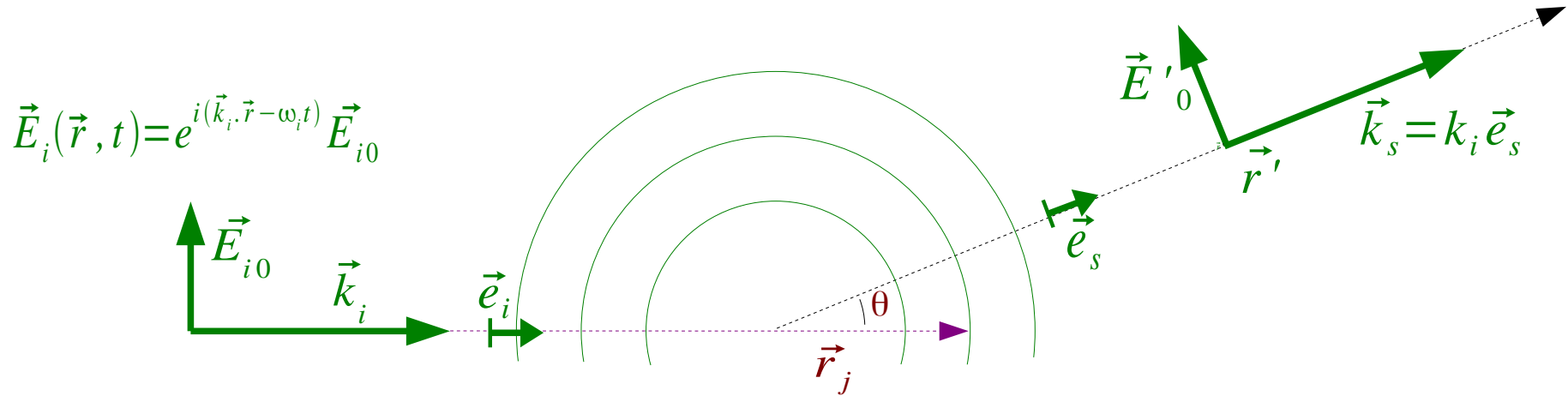
$$d_t \vec{\beta}_j = \frac{-q_e}{m_e C} \vec{E}_i(\vec{r}_j(t), t)$$

Relativistic electron acceleration :

$$d_t \vec{\beta}_j = \frac{-q_e}{m_e \gamma_j C} \left[\vec{E}_i - (\vec{\beta}_j \cdot \vec{E}_i) \vec{\beta}_j + (\vec{\beta}_j \cdot \vec{E}_i) \vec{e}_i - (\vec{\beta}_j \cdot \vec{e}_i) \vec{E}_i \right]$$

$$\gamma_j = \frac{1}{\sqrt{1 - \beta_j^2}}$$

Thomson Scattering : relativistic scattered electric field



Scattered electric as a function particle velocity

$$\vec{E}_{sj}(\vec{r}', t) = \frac{\mu_0 C q_e}{4\pi} \frac{e^{ik_i|\vec{r}' - \vec{r}_j|}}{|\vec{r}' - \vec{r}_j|} \vec{e}_{sj} \wedge (\vec{e}_{sj} \wedge d_t \vec{\beta}_j)$$

$$\vec{\beta}_j(t) = \frac{1}{C} \vec{v}_j(t)$$

Electron scattered field classical expression :

$$\vec{E}_{sclaj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{ik_i|\vec{r}' - \vec{r}_j|}}{|\vec{r}' - \vec{r}_j|} e^{i(\vec{k}_i \cdot \vec{r}_j - \omega_i t)} \vec{e}_{sj} \wedge (\vec{e}_{sj} \wedge \vec{E}_{i0})$$

Electron scattered field relativistic expression :

$$\vec{E}_{srelj}(\vec{r}', t) = \frac{\mu_0 q_e^2}{4\pi m_e} \frac{e^{ik_i|\vec{r}' - \vec{r}_j|}}{|\vec{r}' - \vec{r}_j|} e^{i(\vec{k}_i \cdot \vec{r}_j - \omega_i t)} \frac{1}{(1 - \vec{e}_{sj} \cdot \vec{\beta}_j)^3} \vec{e}_{sj} \wedge [(\vec{e}_{sj} - \vec{\beta}_j) \wedge \vec{E}_{i0}]$$

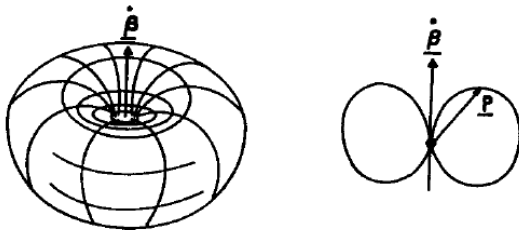
Incoherent Thomson Scattering : relativistic scattered power

Classical scattered cross section power spectrum :

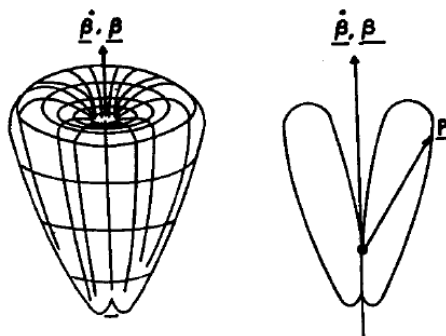
$$\frac{d\sigma}{d\Omega_s}(\theta, \varphi, \omega) = N_s r_0^2 \cos^2 \theta_{pol} \frac{2\pi}{k} f_{ev_k}\left(\frac{-\omega}{k}\right)$$

For the relativistic case, the scattering signal frequency spectrum is not proportional to the velocity distribution (1st order development in β):

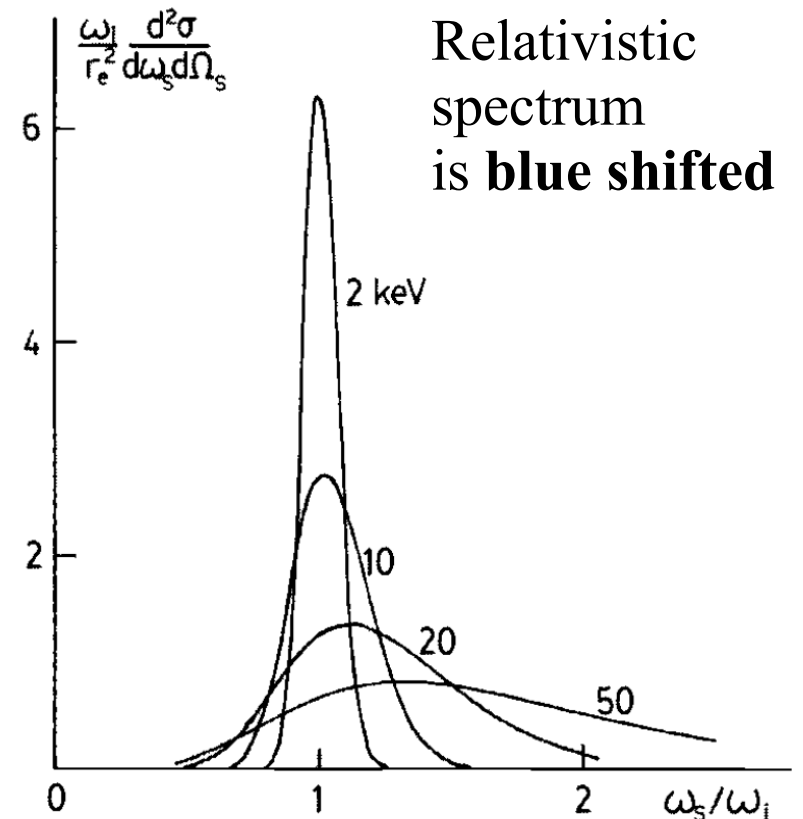
$$\frac{d\sigma_{rel}}{d\Omega_s}(\theta, \varphi, \omega) = N_s r_0^2 \frac{1}{\sqrt{2(1 - \cos \theta_{pol})}} \left(1 + \frac{3\omega}{2\omega_i}\right) \frac{2\pi}{k_i} f_{ev_k}\left(\frac{-\omega}{k}\right)$$



Classical limit



Relativistic case :
enhanced power in the forward direction



Thomson Scattering : magnetized plasma $\omega_i \gg \omega_{ce}$

When $\omega_i \gg \omega_{ce}$ plasma polarization due to the incident wave is not modified by the presence of the magnetic field.

In the presence of the magnetic field, charged particles have cyclotron trajectories. Charged particles do not have rectilinear trajectories.

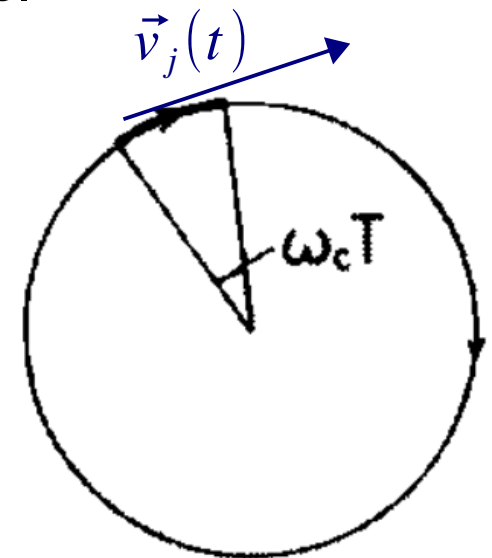
But in the case incident wave frequency is much larger than the electron cyclotron frequency, $\omega_i \gg \omega_{ce}$, we will consider that the scattering signal correlation time is much shorter than the electron cyclotron time $T_{C_{\vec{k}}} \omega_{ce} \ll 1$.

The electron then rotates a small angle along the cyclotron circle.

This movement is almost a rectilinear movement.

The angle position along the cyclotron circle is random.

The scattering signal spectrum will then reproduce the velocity distribution.



$$C_{\vec{k}}(\tau) = \left\langle \sum_j^{N_s} e^{i\vec{k} \cdot \vec{v}_j(t)\tau} \right\rangle_t$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{ev_k} \left(\frac{-\omega}{k} \right)$$

Thomson Scattering : magnetized plasma $\omega_i \sim \omega_{ce}$

Scattering signal correlation expression for a magnetized plasma

$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{-i(\vec{k} \cdot \vec{r}_j(t) - \vec{k} \cdot \vec{r}_j(t+\tau))} \right\rangle_t$$

The phase for each electron is written :

$$\vec{k} \cdot \vec{r}_j(t+\tau) - \vec{k} \cdot \vec{r}_j(t) = \frac{k_{\perp} u_{j\perp}}{\omega_{ce}} \sin(\omega_{ce} \tau) + k_{\parallel} v_{j\parallel} \tau$$

$$\begin{aligned} k_{\parallel} &= \vec{k} \cdot \vec{e}_B & v_{j\parallel} &= \vec{v} \cdot \vec{e}_B \\ \vec{k}_{\perp} &= \vec{k} - k_{\parallel} \vec{e}_B & \vec{v}_{j\perp} &= \vec{v} - v_{j\parallel} \vec{e}_B \end{aligned}$$

We use 1st type Bessel functions to develop sinus function inside the phase :

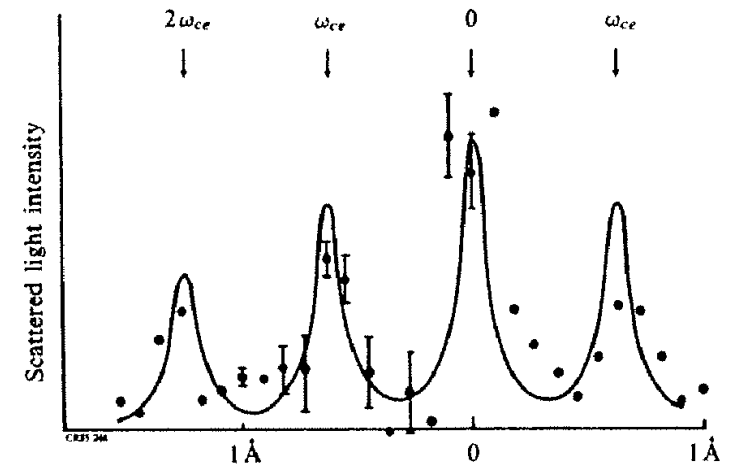
$$e^{i z \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(z) e^{i n \theta}$$

The electron correlation phase is written :

$$e^{i \left[\frac{k_{\perp} u_{j\perp}}{\omega_{ce}} \sin(\omega_{ce} \tau) \right]} = \sum_{n=-\infty}^{\infty} J_n \left(\frac{k_{\perp} u_{j\perp}}{\omega_{ce}} \right) e^{i n \omega_{ce} \tau}$$

$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{i k_{\parallel} v_{j\parallel} \tau} \sum_{n=-\infty}^{+\infty} J_n \left(\frac{k_{\perp} u_{j\perp}}{\omega_{ce}} \right) e^{i n \omega_{ce} \tau} \right\rangle_t$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} \sum_{n=-\infty}^{+\infty} \iiint d^3 \vec{v} J_n \left(\frac{k_{\perp} v_{\perp}}{\omega_{ce}} \right) \delta(\omega + k_{\parallel} v_{\parallel} + n \omega_{ce}) f_{e\vec{v}}(\vec{v})$$



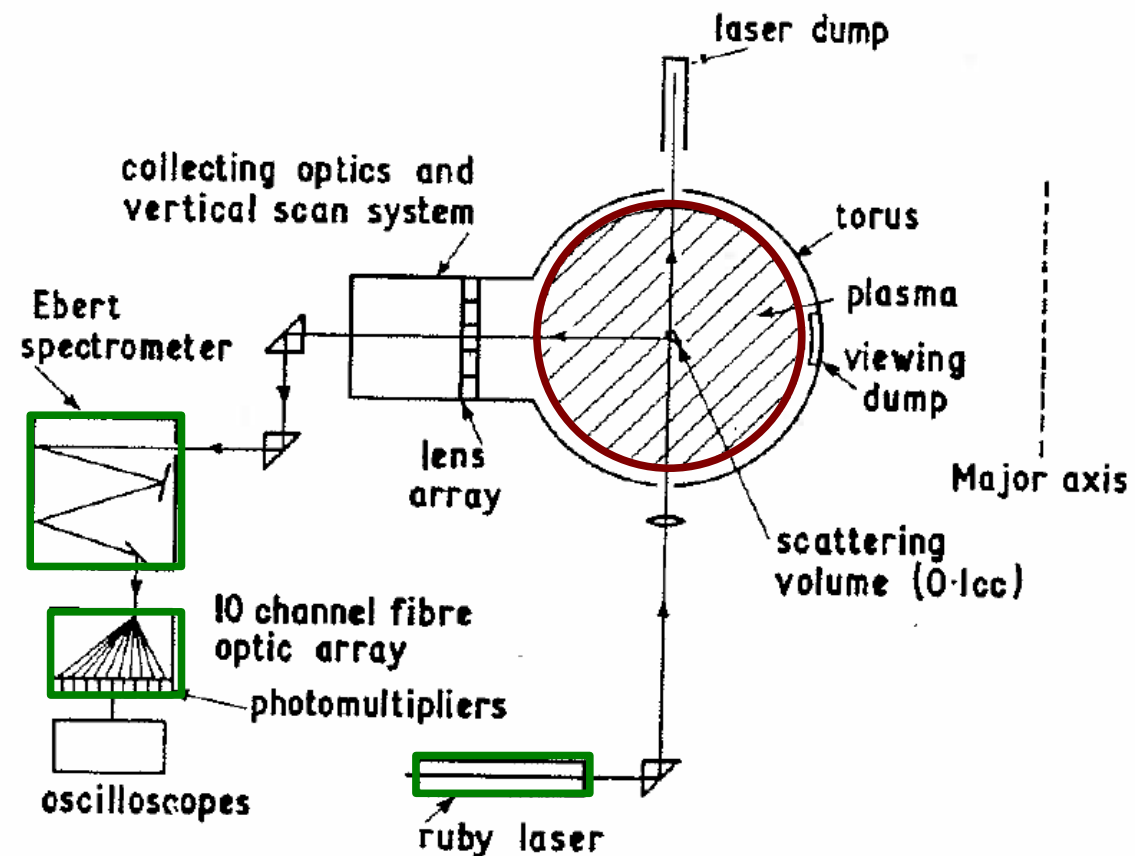
P Carolan & D Evans,
Int. Conf. Ph. Ion. Gases (1971)

Incoherent Thomson Scattering on T3 Tokamak

For first Tokamaks (T1, T3), plasma electron temperature was evaluated using the magnetic measurements and using models connecting the plasma conductivity to the plasma collisionality, and then the electron temperature.

In order to check this evaluation, a British group made Thomson scattering measurements on the Tokamak Plasma.

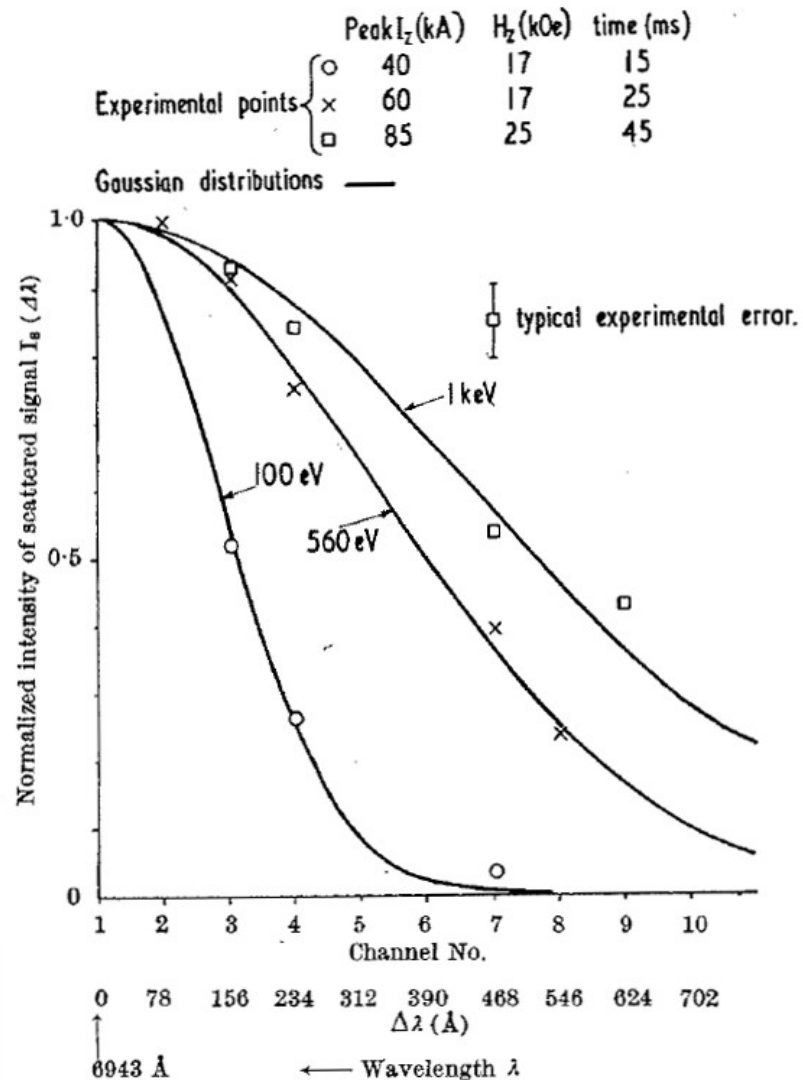
Ruby laser : $\lambda_i = 0.6943 \mu m$



Arstimovich et al., Plasma Physics, **7** (1965), p 305

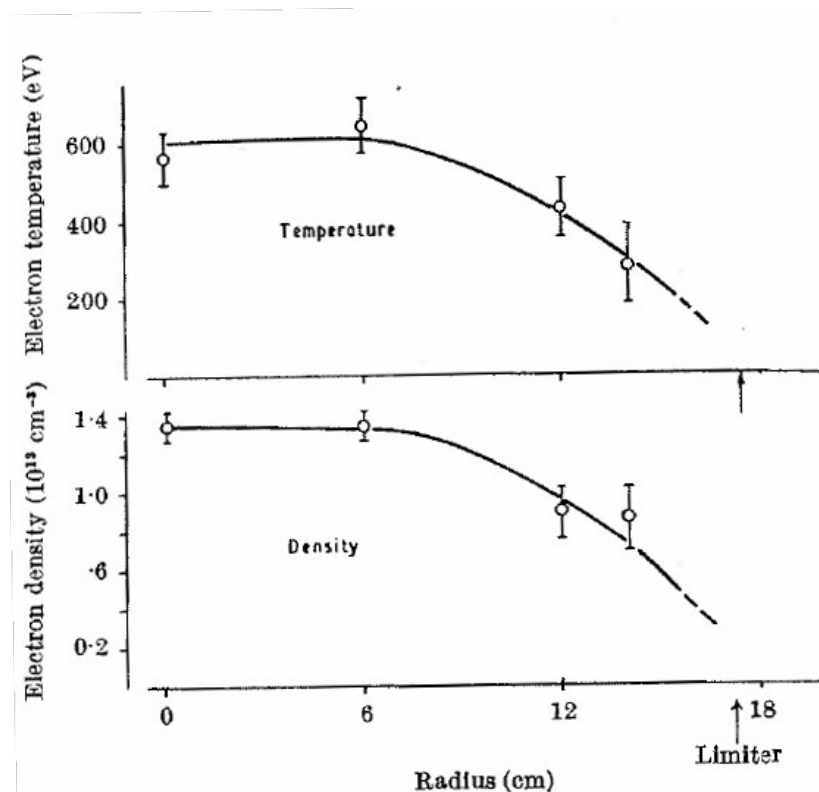
Peacock et al., Nature, **224**, (1/11/1969), p 488

T3 Tokamak electron density and temperature profiles



Scattered power was measured for 10 different frequency shift.

Data are fitted with Maxwellian in order to estimate the temperature.



$$\Delta v_i = \left(-v_i / \lambda_i \right) \Delta \lambda_i \quad \Delta v = v_e / \lambda$$

$$v_i \lambda_i = C = (v_i + \Delta v_i) (\lambda_i + \Delta \lambda_i)$$

Measurements were done at different positions in order to get the electron temperature profile.

LIDAR Incoherent TS on JET

Light Detection And Ranging : short time laser pulse back-scattering measurements. Same measurement principle as radars : localization is function of the time of flight.

The LIDAR only needs one **access port to the device**.

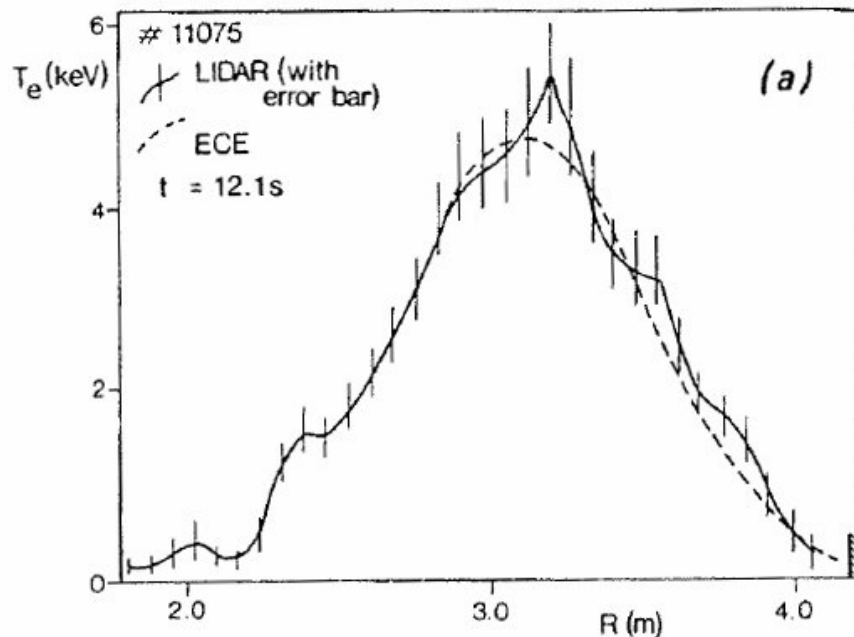
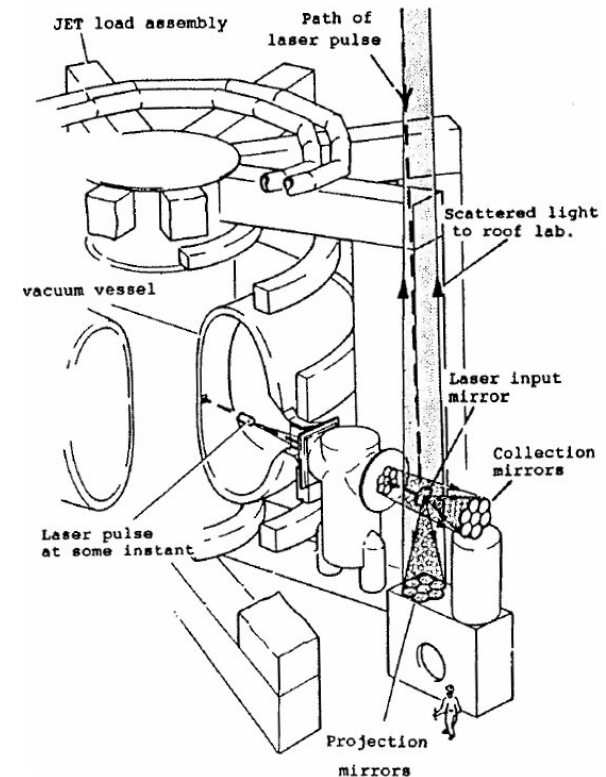
Spatial resolution is proportional to the Laser pulse short duration : we need a more sensitive detector.

A Thomson scattering LIDAR is present on JET :

Pulsed Ruby Laser (694 nm, 2 J, 220 ps, 0,5 Hz)

Polychromator with 6 spectral channels

MCP Photomultiplier



Spatial resolution :

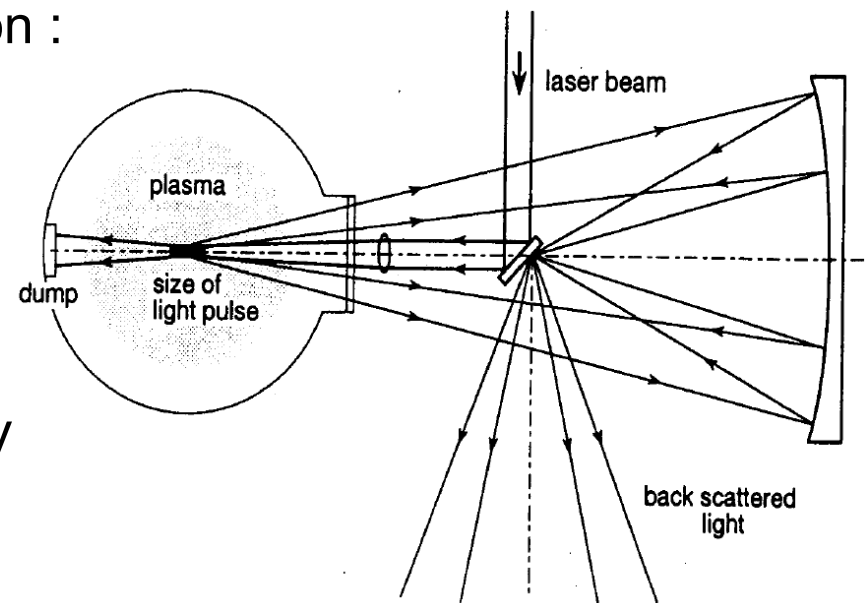
15 cm

Temperature resolution :

± 5 to 20%

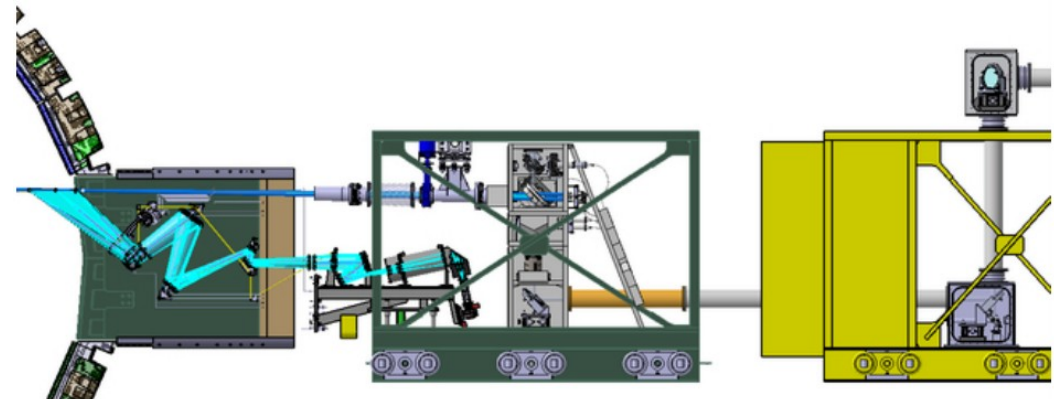
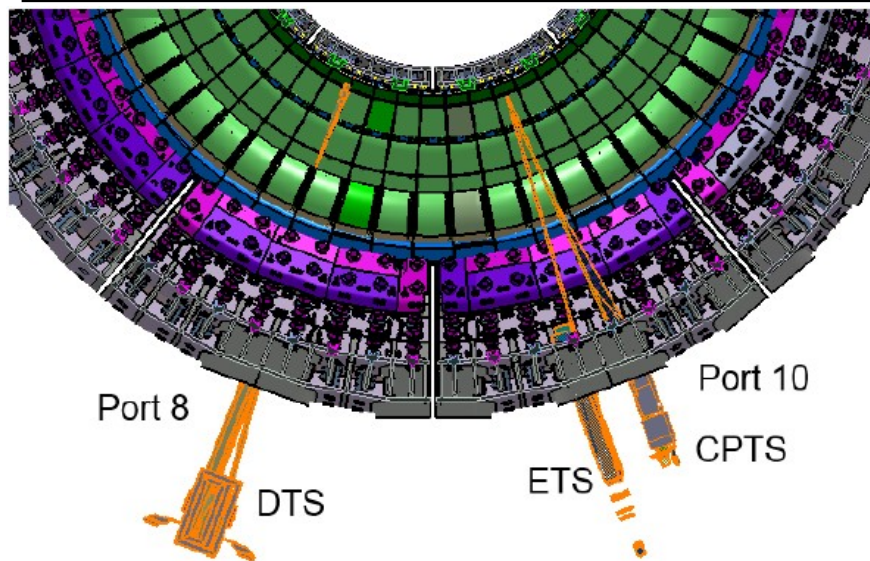
Temperature range :

0.5 to 20 keV



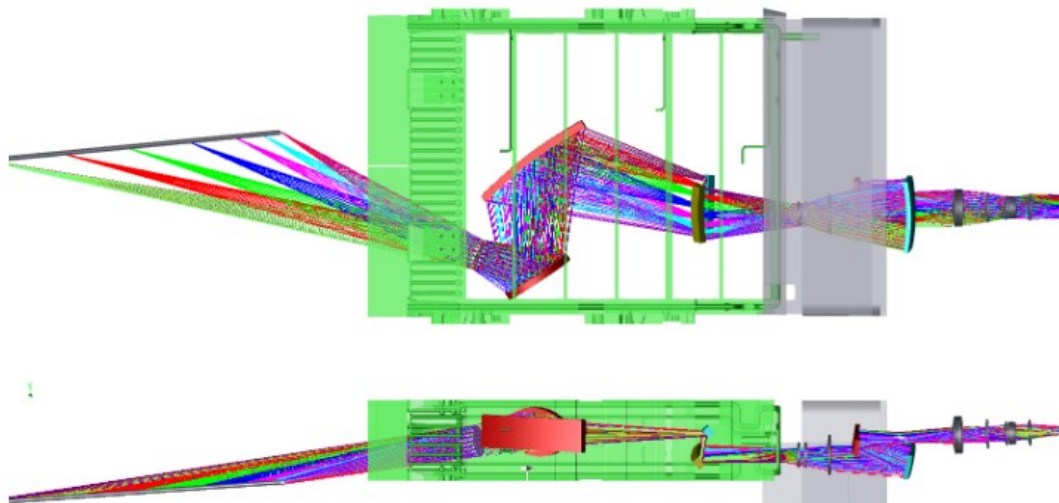
Salzmann et al., Nuclear Fusion, **27**, p 1925 (1987)

ITER incoherent Thomson Scattering

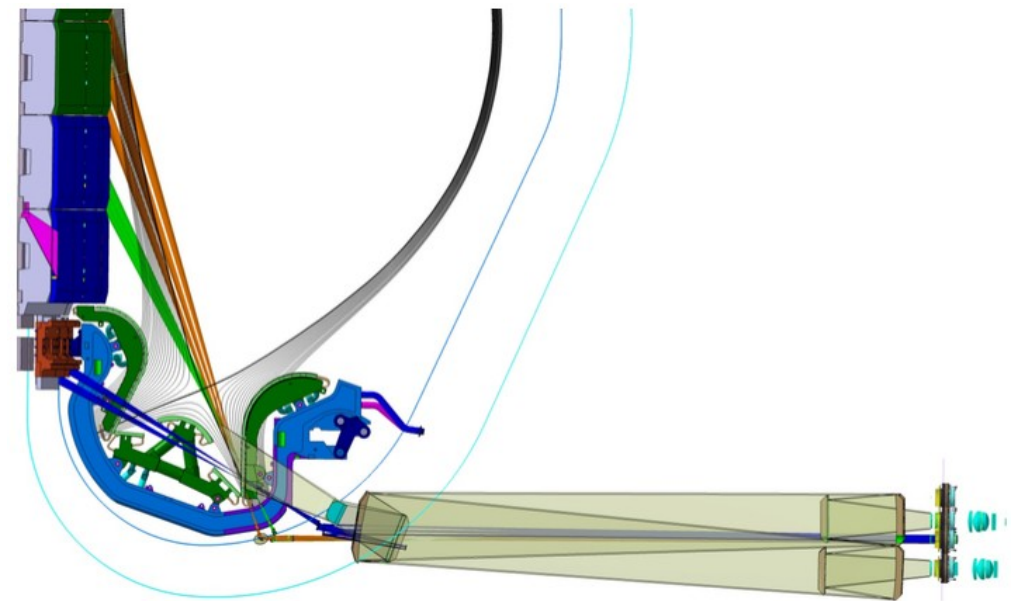


Edge Thomson Scattering system

3 Thomson Scattering systems around ITER



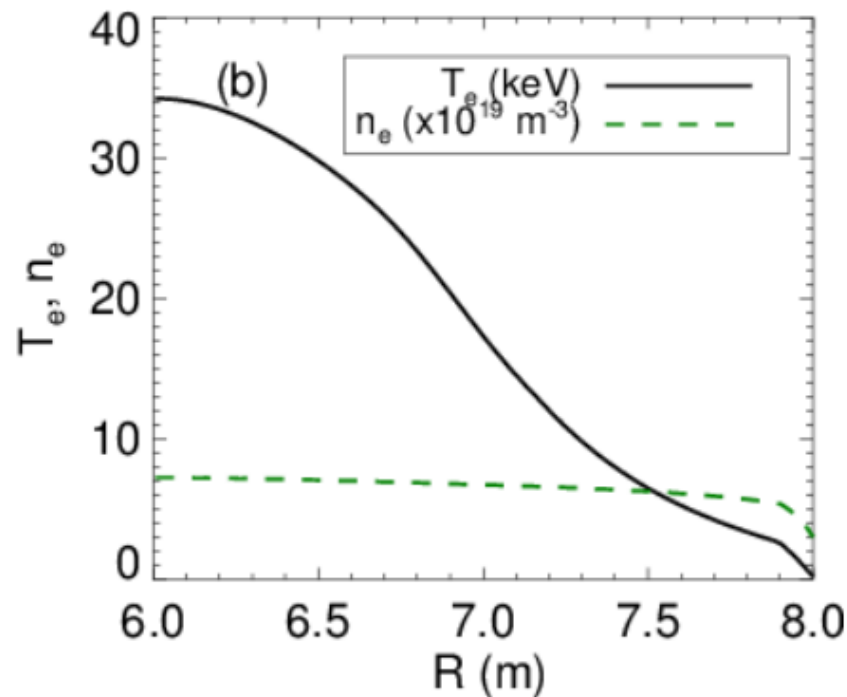
Core Plasma Thomson Scattering system



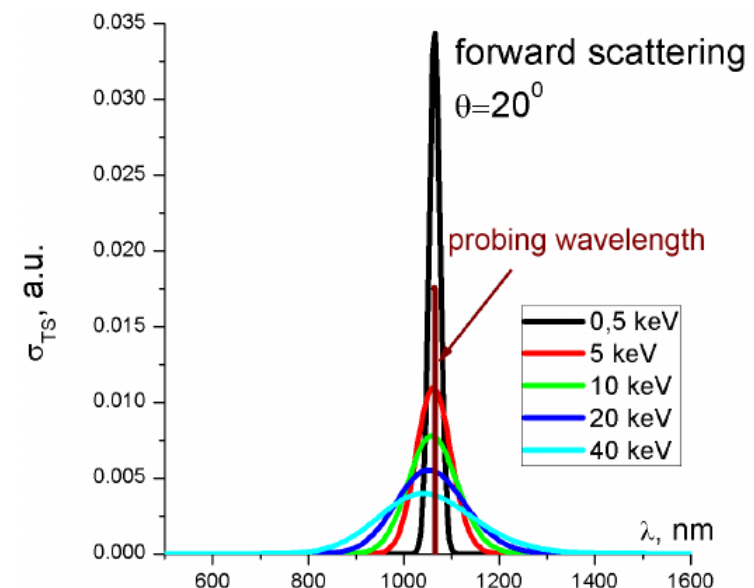
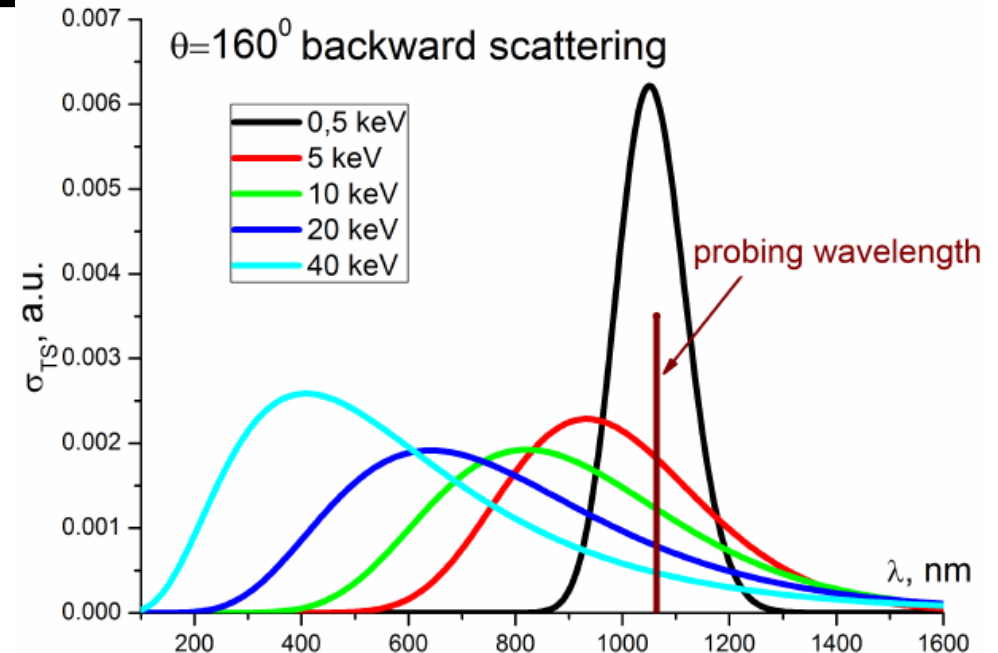
Divertor Thomson Scattering system

Bassan et al., J. Instr., **11**, C01052 (2016)

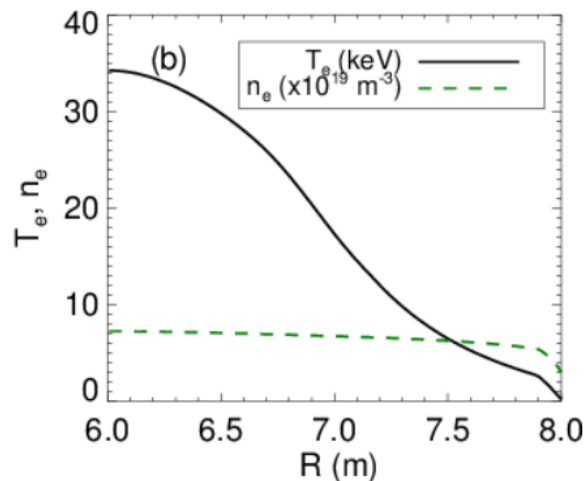
ITER incoherent Thomson Scattering



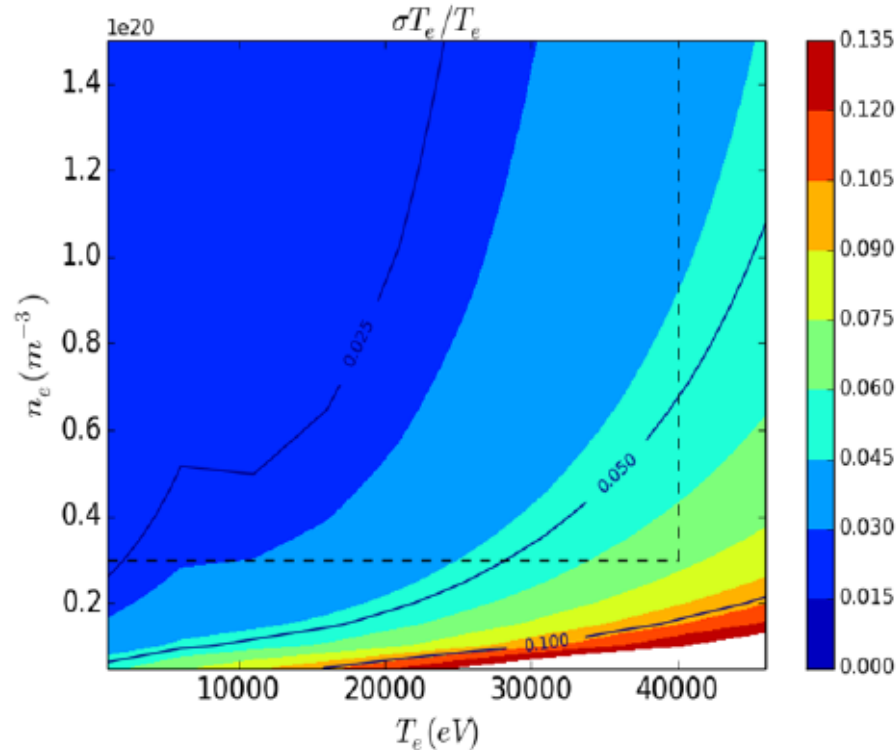
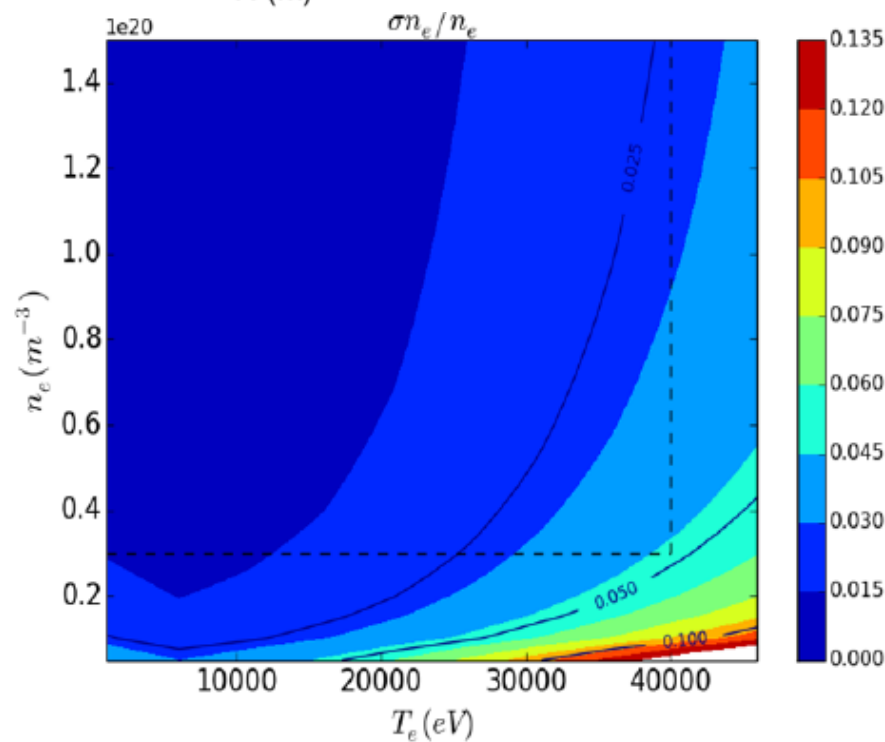
Simulated Thomson scattering cross-section depending on the scattering direction and electron temperature.



ITER incoherent Thomson Scattering



Electron density and temperature measurement accuracy depends on their value in the plasma



Scannell et al., J. Instr., **12**, C11010 (2017)

Coherent Thomson Scattering

Coherent Thomson Scattering

The Scattering Signal is the sum of all scatterer scattering phases :

$$n_{\vec{k}}(t) = \sum_j^{N_s} e^{-i\vec{k} \cdot \vec{r}_j(t)}$$

The sum can be replaced with the Klimontovich description of the density :

$$n_{KL}(\vec{r}, t) = \sum_j \delta(\vec{r} - \vec{r}_j(t))$$

$$n_{\vec{k}}(t) = \iiint_{V_s} n_{KL}(\vec{r}, t) e^{-i\vec{k} \cdot \vec{r}} d^3 \vec{r}$$

When the scattering wavelength $\lambda = 2\pi/k$ is large enough, $k\lambda_D < 1$ $\lambda > 2\pi\lambda_D$ we switch from the microscopic description to a mesoscopic description :

$$n_{KL}(\vec{r}, t) \rightarrow n_e(\vec{r}, t)$$

For a mesoscopic point of view, the scattering signal is the spatial Fourier transform of the electron density :

$$n_{\vec{k}}(t) = \iiint_{V_s} n_e(\vec{r}, t) e^{-i\vec{k} \cdot \vec{r}} d^3 \vec{r}$$

Coherent Thomson Scattering

Because of the electron mass, plasma **electrons** are the main **scatterer** particles for Thomson scattering.

But for scattering wavelength larger than the Debye length ($k\lambda_D < 1$), the **electron dynamics** are affected by **ion dynamics** due to the screening by electrons.

Electron density fluctuations are linked also to ion density fluctuations.

The plasma will be considered as a **dielectric medium**.

Plasma susceptibility

Expression of the **electric displacement** for a dielectric medium :
the electric field is modified by the presence of **dipoles** :

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

The dipole field is function of the plasma **susceptibility** :

$$\vec{P} = \chi \epsilon_0 \vec{E}$$

The plasma response is the sum of the electron and the ion response :

$$\vec{P} = \vec{P}_e + \vec{P}_i \quad \vec{P}_e = \chi_e \epsilon_0 \vec{E} \quad \vec{P}_i = \chi_i \epsilon_0 \vec{E}$$

$$\vec{D} = \epsilon_0 (1 + \chi_e + \chi_i) \vec{E} \quad \epsilon_r = 1 + \chi_e + \chi_i$$

The medium electron charge due to the electric displacement :

$$\rho_e = -\vec{\nabla} \cdot \vec{P}_e = \frac{-\chi_e}{1 + \chi_e + \chi_i} \vec{\nabla} \cdot \vec{D}$$

We have to determine $\vec{D}$ in order to know the electron density fluctuations used for scattering signal :

$$n_e = \frac{-1}{q_e} \rho_e$$

Coherent Thomson Scattering: shielding effect

For any **free test particle** with charge q_n and velocity $\vec{v}_n$,
The charge dynamics is :

$$\rho_n(\vec{x}, t) = q_n \delta(\vec{x} - \vec{v}_n t)$$

The Fourier transform of this moving charge is :

$$\rho_n(\vec{k}, \omega) = 2\pi q_n \delta(\vec{k} \cdot \vec{v}_n + \omega)$$

$$\rho_n(\vec{k}, \omega) = \iiint d^3\vec{r} \int dt \rho_n(\vec{r}, t) e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

This test particle generates an **electric displacement** field :

$$\rho_n = \vec{\nabla} \cdot \vec{D}$$

Gauss law for free test particle

So the test particle modifies the electron charge distribution through the **screening effect** :

$$\rho_e(\vec{k}, \omega) = \frac{-\chi_e}{1 + \chi_e + \chi_i} 2\pi q_n \delta(\vec{k} \cdot \vec{v}_n + \omega)$$

$$\rho_e = \frac{-\chi_e}{1 + \chi_e + \chi_i} \vec{\nabla} \cdot \vec{D}$$

The electron charge distribution, will be the sum of the electron distribution,
the electron shielding electron distribution and the electron shielding ion distribution

The resulting electric charge distribution is modified by all test particles :

$$\tilde{\rho}_e(\vec{k}, \omega) = \sum_n^{N_{part}} \frac{-\chi_e}{1 + \chi_e + \chi_i} 2\pi q_n \delta(\vec{k} \cdot \vec{v}_n + \omega)$$

$$n_e(\vec{k}, \omega) = \frac{-1}{q_e} \rho_e(\vec{k}, \omega)$$

Z_i : ion charge number

$$n_e(\vec{k}, \omega) = \frac{-1}{q_e} \sum_j^{N_{el}} \left[1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right] 2\pi (-q_e) \delta(\vec{k} \cdot \vec{v}_j + \omega) + \frac{-1}{q_e} \sum_l^{N_{ion}} \left[\frac{-\chi_e}{1 + \chi_e + \chi_i} \right] 2\pi Z_i q_e \delta(\vec{k} \cdot \vec{v}_l + \omega)$$

Coherent Thomson Scattering: the form factor

Electron distribution corrected by the shielding effect :

$$n_e(\vec{k}, \omega) = \frac{-1}{q_e} \sum_j^{N_{el}} \left[1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right] 2\pi (-q_e) \delta(\vec{k} \cdot \vec{v}_j + \omega) + \frac{-1}{q_e} \sum_l^{N_{ion}} \left[\frac{-\chi_e}{1 + \chi_e + \chi_i} \right] 2\pi Z_i q_e \delta(\vec{k} \cdot \vec{v}_l + \omega)$$

Dynamical form factor :

$$S_{\vec{k}}(\omega) = \lim_{T \rightarrow +\infty} \frac{1}{T N_s} |n_{eT}(\vec{k}, \omega)|^2 \quad n_{eT}(\vec{k}, \omega) = \int_t^{t+T} dt n_e(\vec{k}, t) e^{i\omega t}$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{N_s} \left\langle \sum_j^{N_{el}} \left[1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right]^2 \delta(\vec{k} \cdot \vec{v}_j + \omega) + \sum_l^{N_{ion}} \left[\frac{-\chi_e}{1 + \chi_e + \chi_i} \right]^2 Z_i^2 \delta(\vec{k} \cdot \vec{v}_l + \omega) \right\rangle_T$$

For each species l , $F_{\vec{v}_l}$ is the velocity distribution :

$$\left\langle \sum_j^{N_l} \delta(\vec{k} \cdot \vec{v}_j + \omega) \right\rangle_T = \iiint_{V_s} d^3 \vec{r} \iiint d^2 \vec{v} F_{\vec{v}_l}(\vec{v}) \delta(\vec{k} \cdot \vec{v} + \omega) \quad \text{Ergodic hypothesis}$$

$f_{v_{kl}}$ is the 1D velocity distribution along $\vec{k}$

$$\left\langle \sum_j^{N_l} \delta(\vec{k} \cdot \vec{v}_j + \omega) \right\rangle_T = V_s \int dv_k f_{v_{kl}}(v_k) \delta(k v_k + \omega)$$

$$\left\langle \sum_j^{N_l} \delta(\vec{k} \cdot \vec{v}_j + \omega) \right\rangle_T = V_s \frac{1}{k} f_{v_{kl}}\left(\frac{-\omega}{k}\right)$$

The form factor expression :

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} \left[\left| 1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right|^2 f_{v_{ke}}\left(\frac{-\omega}{k}\right) + \left| \frac{-\chi_e}{1 + \chi_e + \chi_i} \right|^2 Z_i^2 f_{v_{ki}}\left(\frac{-\omega}{k}\right) \right]$$

Coherent Thomson Scattering: susceptibility

The form factor expression :

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} \left[\left| 1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right|^2 f_{v_k e} \left(\frac{-\omega}{k} \right) + \left| \frac{-\chi_e}{1 + \chi_e + \chi_i} \right|^2 Z_i^2 f_{v_k i} \left(\frac{-\omega}{k} \right) \right] \quad k\lambda_D < 1$$

The dynamical form factor does not depend only on electron velocity distribution, but also on **each ion species velocity distribution**.

When we neglect the electron screening for ions and other electrons, $\chi_e = 0$ and all $\chi_i = 0$ we obtain the expression for **incoherent** Thomson scattering :

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{v_k e} \left(\frac{-\omega}{k} \right)$$

What are the **values** of χ_e and χ_i ?

For cold plasmas longitudinal waves :

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \quad \epsilon_r = 1 + \chi \quad \chi = -\frac{\omega_p^2}{\omega^2}$$

These expressions apply to the incident beam characteristics $\vec{k}_i, \omega_i$

They do not apply for waves corresponding to thermal plasma particle velocities. $\vec{k}, \omega_D = k v_{Te}$

We need to apply kinetic theory : the wave electric field modifies the each species velocity distribution.

Coherent Thomson Scattering: susceptibility

Using Vlasov equation :

$$\partial_t f_{\iota} + \vec{v} \cdot \vec{\nabla}_{\vec{r}} f_{\iota} + \frac{q_{\iota}}{m_{\iota}} \vec{E} \cdot \vec{\nabla}_{\vec{v}} f_{\iota} = 0$$

$f_{\iota 0}$ is the initial particle position and velocity distribution.

$f_{\iota 1}(\vec{k}, \omega)$ is the response to an external Fourier mode perturbation : $\vec{E}_1(\vec{k}, \omega)$

The Fourier transform of the 1st order Vlasov equation is :

$$-i\omega f_{\iota 1} + i\vec{k} \cdot \vec{v} f_{\iota 1} + \frac{q_{\iota}}{m_{\iota}} \vec{E}_1 \cdot \vec{\nabla}_{\vec{v}} f_{\iota 0} = 0$$

$$f_{\iota 1} = \frac{q_{\iota}}{i m_{\iota}} \frac{\vec{E}_1 \cdot \vec{\nabla}_{\vec{v}} f_{\iota 0}}{\omega - \vec{k} \cdot \vec{v}}$$

An electric current $\vec{j}_{\iota 1}$ is created by the $\vec{E}_1$ electric field :

$$\vec{j}_{\iota 1} = \iiint q_{\iota} \vec{v} f_{\iota 1} d^3 \vec{v} \quad \Rightarrow \quad \vec{j}_{\iota 1} = \frac{q_{\iota}^2}{i m_{\iota}} \iiint \frac{\vec{v} (\vec{E}_1 \cdot \vec{\nabla}_{\vec{v}} f_{\iota 0})}{\omega - \vec{k} \cdot \vec{v}} d^3 \vec{v}$$

The current is the dipole time derivative :

$$\vec{j}_{\iota 1} = \partial_t \vec{P}_{\iota 1} \quad \vec{P}_{\iota 1} = \epsilon_0 \chi_{\iota} \cdot \vec{E}_1 \quad \Rightarrow \quad \vec{j}_{\iota 1} = -i\omega \epsilon_0 \chi_{\iota} \cdot \vec{E}_1$$

$$\text{Hence : } \chi_{\iota}(\vec{k}, \omega) = \frac{q_{\iota}^2}{m_{\iota} \epsilon_0 k} \int \frac{\partial_v f_{\iota k 0}}{\omega - \vec{k} \cdot \vec{v}_k} d v_k$$

The **conductivity** tensor connect the electric field to the current : $\vec{j}_{\iota 1} = \bar{\bar{\sigma}}_{\iota} \cdot \vec{E}_1$

We integrate the velocity distribution for the 2 directions perpendicular to $\vec{k}$

We assume the initial distribution is **isotropic** (no external magnetic field).

The species **susceptibility** is connected to their conductivity : $\chi_{\iota} = -\frac{\sigma_{\iota}}{i\omega \epsilon_0}$

Coherent Thomson Scattering : Maxwellian distribution

Susceptibility for each species :

$$\chi_i = \frac{q_i^2}{m_i \epsilon_0 k} \int \frac{\partial_v f_{v_k i 0}}{\omega - k v_k} d v_k$$

This is a Landau damping like improper integral

For a Maxwellian distribution :

$$f_{v_k i 0}(v_k) = n_{i0} \frac{1}{\sqrt{2\pi}} \left(\frac{1}{v_{Ti}} \right) e^{-\frac{v_k^2}{2v_{Ti}^2}} \quad v_{Ti} = \sqrt{\frac{k_B T_i}{m_i}}$$

According to Fried & Comte :

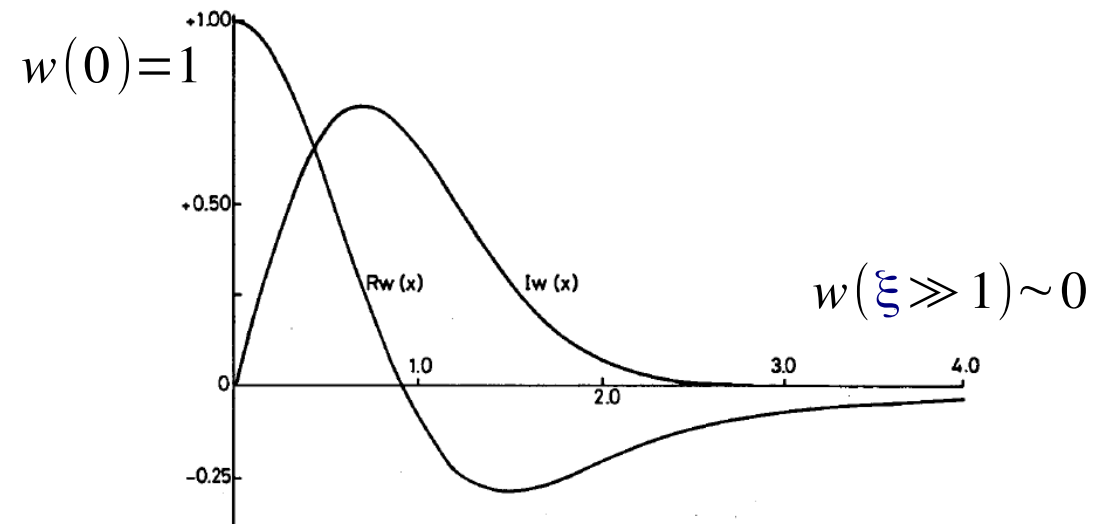
$$\chi_i = \frac{1}{k^2 \lambda_{Di}^2} w\left(\frac{\omega}{k v_{Ti} \sqrt{2}}\right) \quad \xi_i = \frac{\omega}{k v_{Ti} \sqrt{2}}$$

$$\lambda_{Di} = \sqrt{\frac{\epsilon_0 k_B T_i}{n_i q_i^2}} = \frac{v_{Ti}}{\omega_{pi}}$$

w is the plasma **dispersion function** :

$$w(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{\zeta^2} d\zeta + i\sqrt{\pi}\xi e^{-\xi^2}$$

The complex part of the dispersion function corresponds to the Landau Damping.



Coherent Thomson Scattering : Salpeter approximation

Susceptibility for each species :

$$\chi_i = \frac{1}{k^2 \lambda_{Di}^2} w(\xi_i)$$

$$\xi_i = \frac{\omega}{k v_{Ti} \sqrt{2}}$$

$$w(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{\zeta^2} d\zeta + i\sqrt{\pi}\xi e^{-\xi^2}$$

Since $m_e \ll m_i$ and for most cases $T_e \sim T_i$, electron Doppler frequencies are much larger than the ion Doppler frequencies :
 $k v_{Ti} \ll k v_{Te}$

For ω in the range of electron Doppler frequencies : $\omega \sim k v_{Te} \sqrt{2}$

$$\xi_i = \frac{\omega}{k v_{Ti} \sqrt{2}} \gg 1$$

Limit for the plasma dispersion relation:

$$w(\xi_i) \ll 1$$

Ion susceptibility is weak:

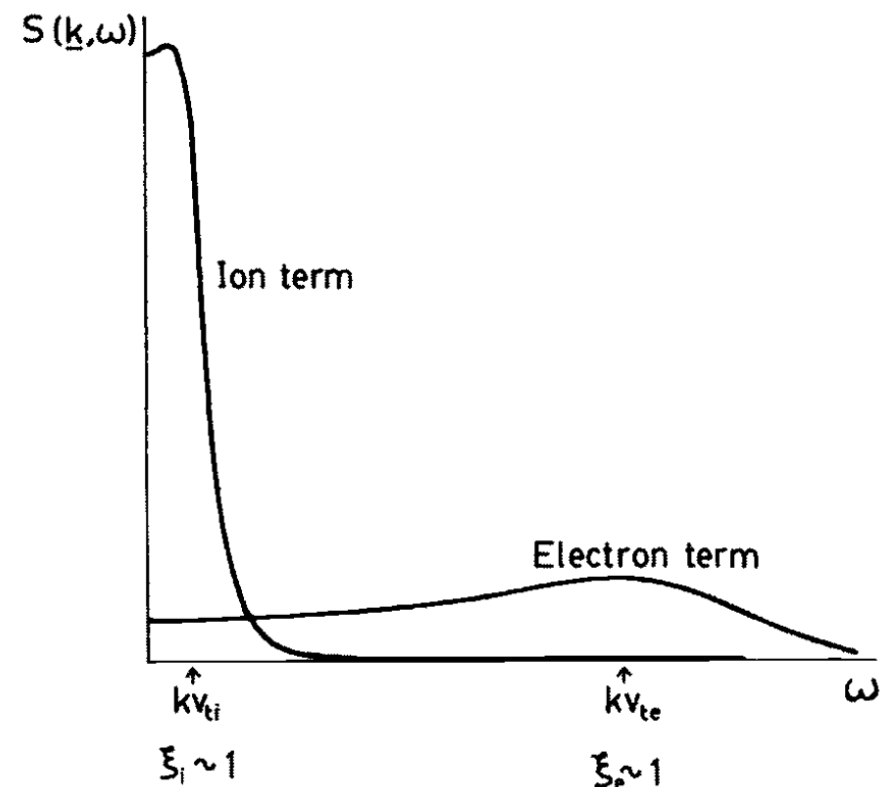
$$\chi_i \ll 1$$

Ion velocity distribution factor is small :

$$f_{v_{ki}}\left(\frac{-\omega}{k}\right) \propto e^{-\xi_i^2} \ll 1$$

The electron term is dominant :

$$S_{\vec{k}}(\omega) \sim \frac{2\pi}{k} \left| \frac{1}{1 + \chi_e} \right|^2 f_{v_{ke}}\left(\frac{-\omega}{k}\right)$$



Coherent Thomson Scattering : Salpeter approximation

Susceptibility for each species :

$$\chi_i = \frac{1}{k^2 \lambda_{Di}^2} w(\xi_i)$$

$$\xi_i = \frac{\omega}{k v_{Ti} \sqrt{2}}$$

$$w(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{(\xi^2)} d\xi + i\sqrt{\pi}\xi e^{-\xi^2}$$

For ω in the range of ion Doppler frequencies : $\omega \sim k v_{Ti} \sqrt{2}$

For the electrons : $\xi_e \ll 1$

The dispersion function factor is close to 1:

$$w(\xi_e) \sim 1$$

The electron susceptibility is almost constant:

$$\chi_e = \alpha^2 \quad \alpha = \frac{1}{k \lambda_D}$$

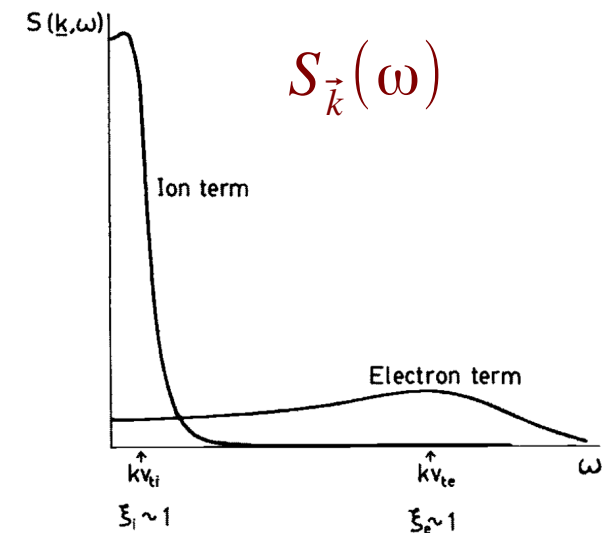
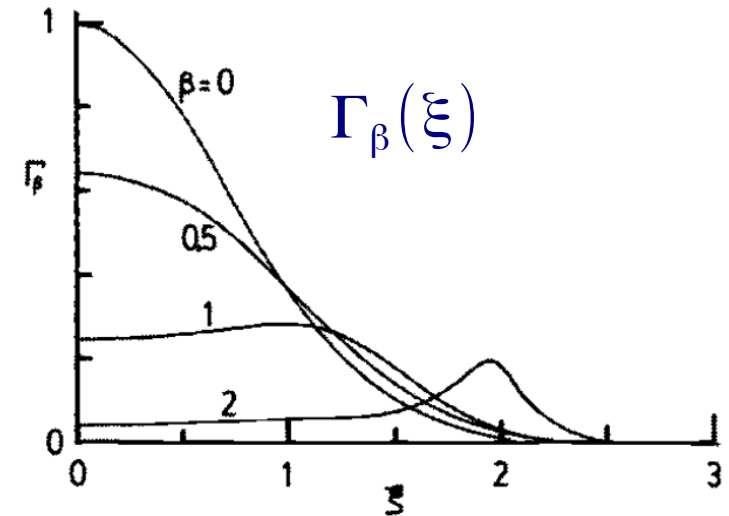
For the ions : $\xi_i \sim 1$

We define the Salpeter function :

$$\Gamma_\beta(\xi) = \frac{e^{-\xi^2}}{|1 + \beta^2 w(\xi)|^2} \quad \beta^2 = \frac{1}{k^2 \lambda_D^2} \frac{T_e}{T_i}$$

The form factor includes electron and ion terms :

$$S_{\vec{k}}(\omega) \sim \frac{\sqrt{2\pi}}{k v_{Te}} \left(\frac{1}{1 + \alpha^2} \right)^2 e^{-\xi_e^2} + \frac{\sqrt{2\pi}}{k v_{Ti}} Z_i^2 \left(\frac{\alpha^2}{1 + \alpha^2} \right)^2 \frac{e^{-\xi_i^2}}{|1 + \beta^2 w(\xi_i)|^2}$$



Coherent Thomson Scattering : practical application

Coherent Thomson scattering needs the condition : $k \lambda_D < 1$

Salpeter approximation : $\omega \sim k v_{Ti} \sqrt{2}$

$$\alpha = \frac{1}{k \lambda_D} \quad \beta^2 = \frac{1}{k^2 \lambda_D^2} \frac{T_e}{T_i}$$

$$S_{\vec{k}}(\omega) = \frac{\sqrt{2} \pi}{k v_{Te}} \left(\frac{1}{1 + \alpha^2} \right)^2 e^{-\xi_e^2} + \frac{\sqrt{2} \pi}{k v_{Ti}} Z_i^2 \left(\frac{\alpha^2}{1 + \alpha^2} \right)^2 \frac{e^{-\xi_i^2}}{|1 + \beta^2 w(\xi_i)|^2} \quad \xi_i = \frac{\omega}{k v_{Ti} \sqrt{2}}$$

The scattering dynamical form factor does not reproduce directly the velocity distribution :
the ion temperature estimation needs model fitting on measurements from multiple spectral channels.

We need to use small angle forward scattering or long wavelength incident beam.

Small angle forward scattering :

- difficult to separate scattered field from incident beam.

Long wavelength sources :

- long wavelength laser
- microwave sources

Multiple ions species might be present : it is difficult to separate them :

- they have to have quite different masses (heavy impurities with High Z)
- they have to have quite different temperatures (helium as fusion product)

Scattering signal interpretation is not obvious because of the Salpeter approximation expression.
Contrary to incoherent Thomson scattering, the scattered signal does not reproduce directly the velocity distribution.

Coherent Thomson Scattering on TEXTOR

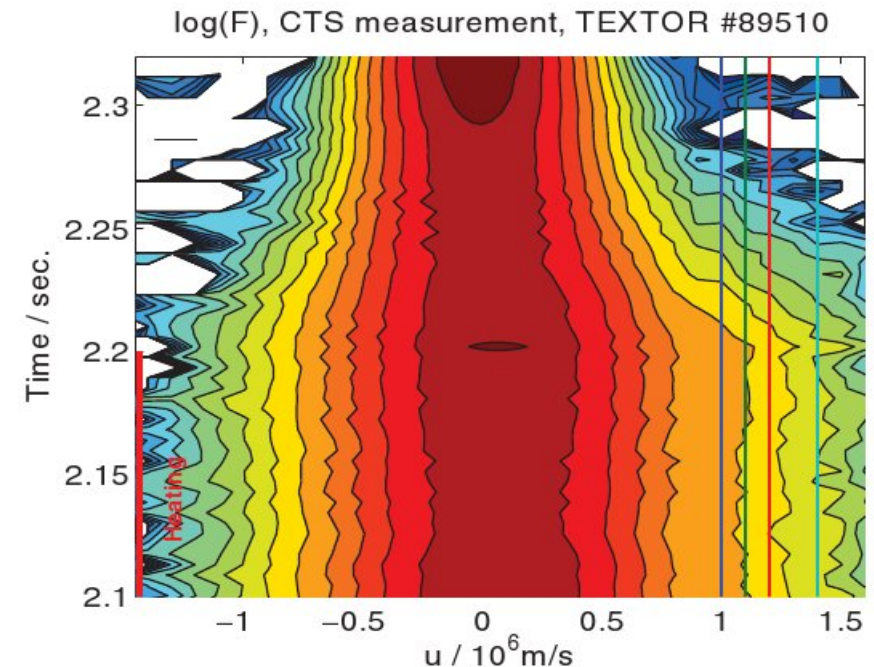
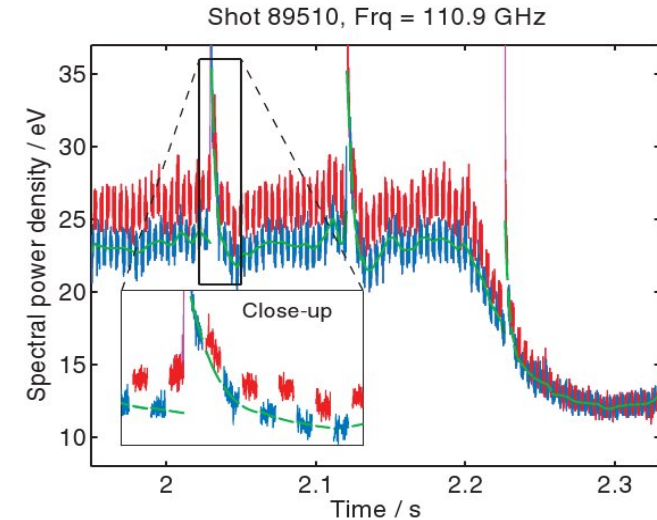
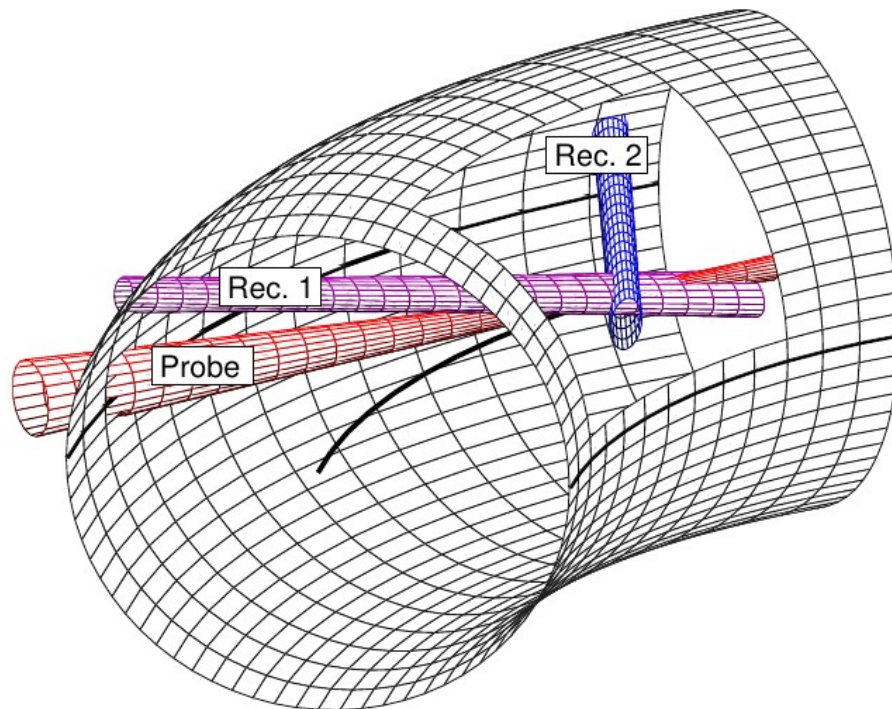
Gyrotron 110 GHz, 150 kW

Pulsed source (4 ms ON, 4 ms OFF) to extract the signal from the ECE

Spatial resolution : 5 to 10 cm

42 spectral channels from 107 to 113 GHz

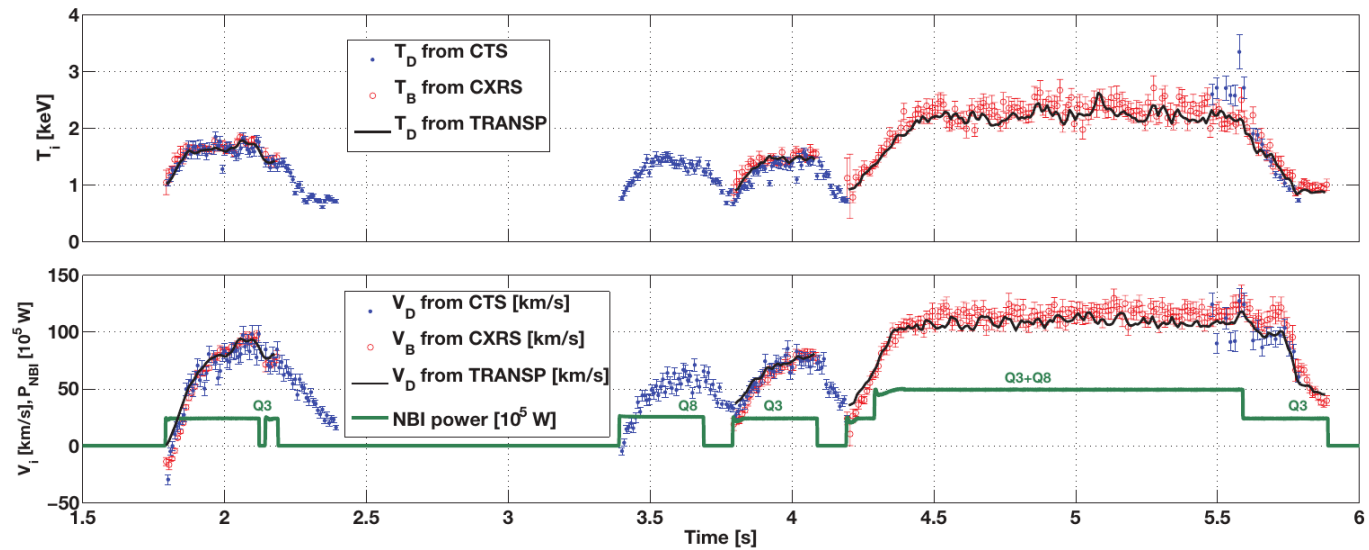
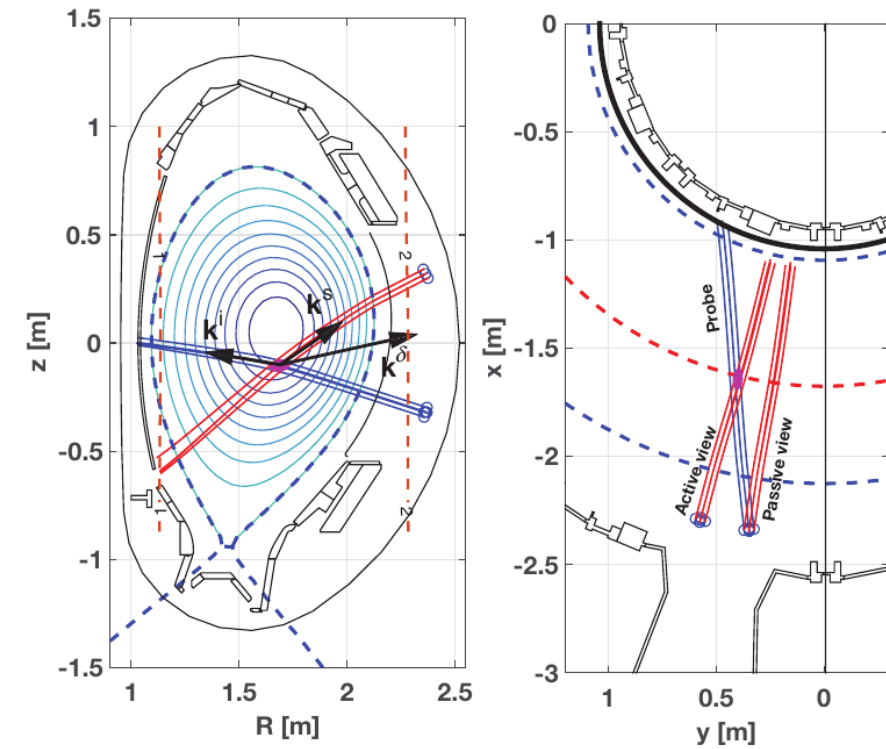
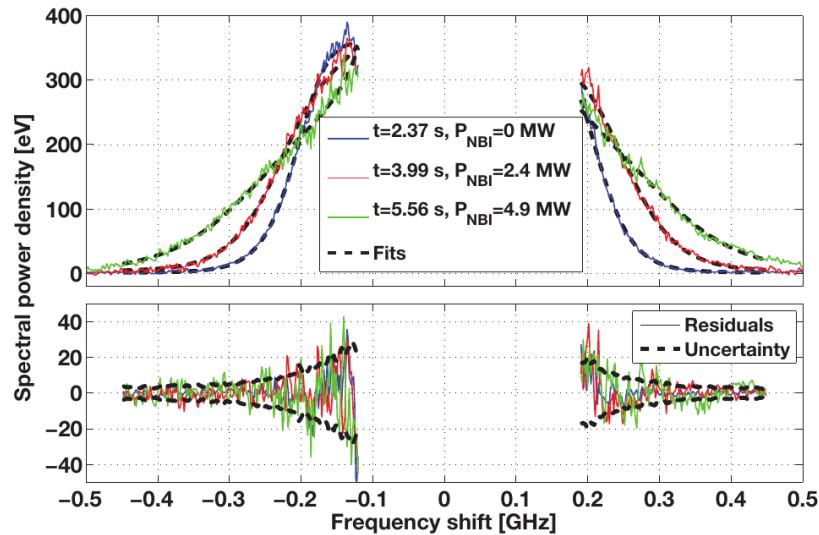
Plasma shot with heating (Neutral Beam Injection) (between $t = 2.1$ and $t = 2.2$ s)



Bindsev et al., Physical Review Letters, **97**, 205005 (2006)

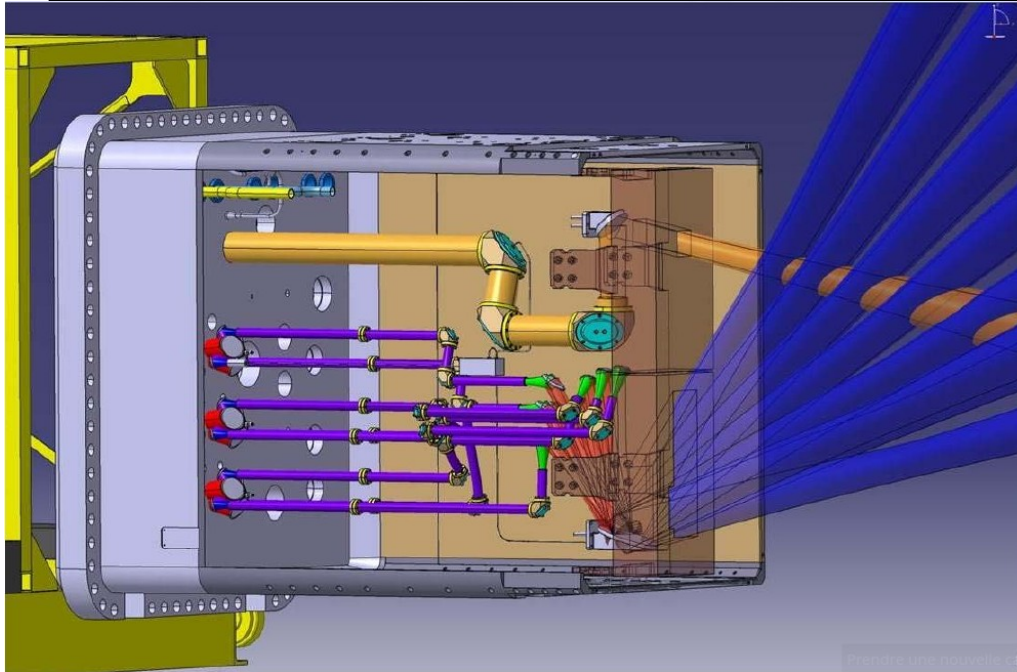
CTS on ASDEX-U

Source : cyclotrons used for ECRH,
105 GHz, 500 kW, 2 ms ON, 8 ms OFF

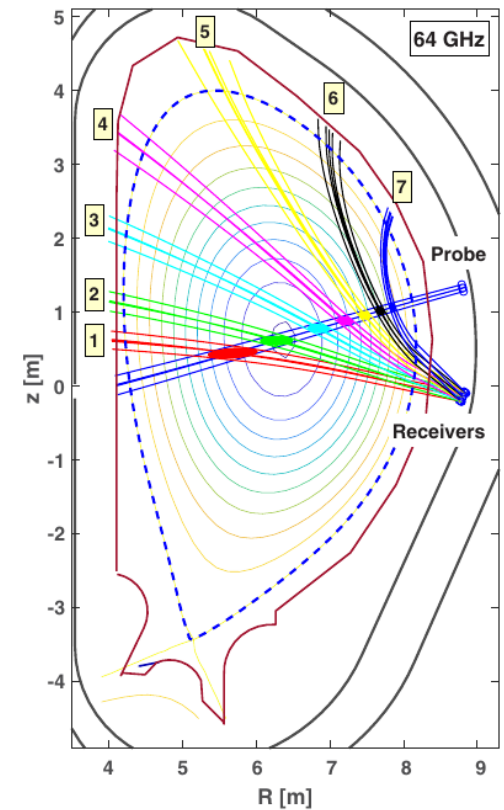
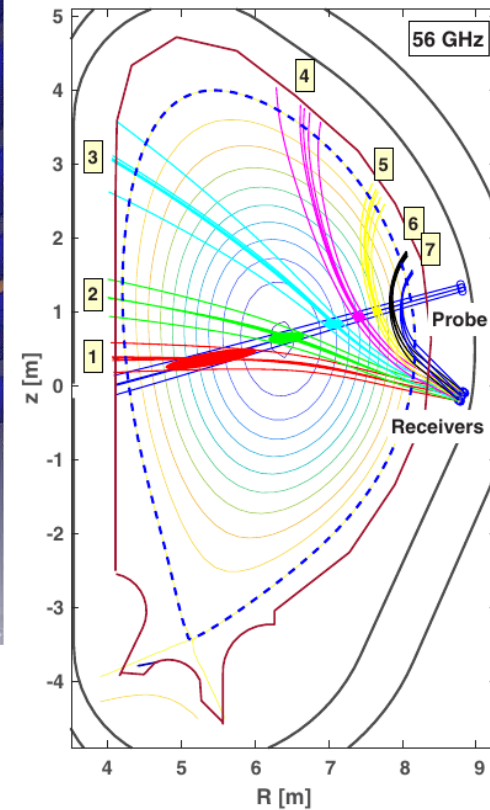


M. Stejner et al., Plasma Physics and Controlled Fusion, **57**, 062001 (2015)

ITER Coherent Thomson Scattering



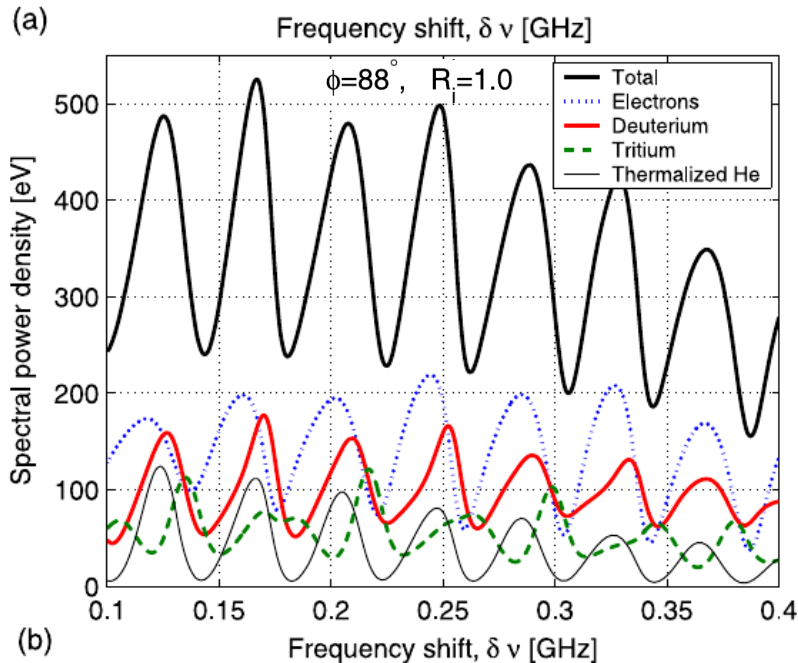
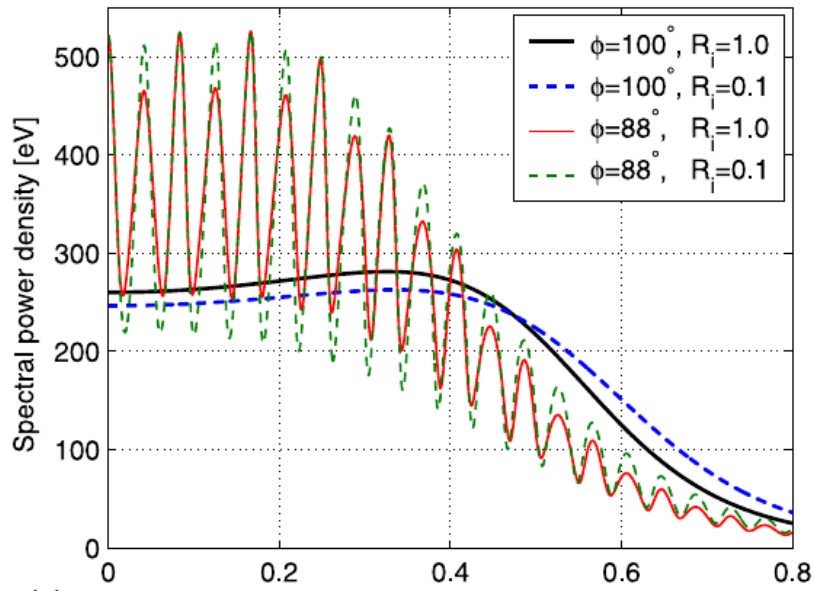
Source : cyclotrons
60 GHz, 1 MW, X mode, 0.1 s resolution



Korsholm et al., IEEE 41st Intern. Conf IRMMW-THz (2016)

Rasmussen et al., Nucl. Fusion, 59, 096031 (2019)

ITER Coherent Thomson Scattering : frequency spectra



Simulated Coherent Thomson scattering spectral power density for different scattering angle and different fuel ion ratio.

Ion cyclotron frequencies intervene.

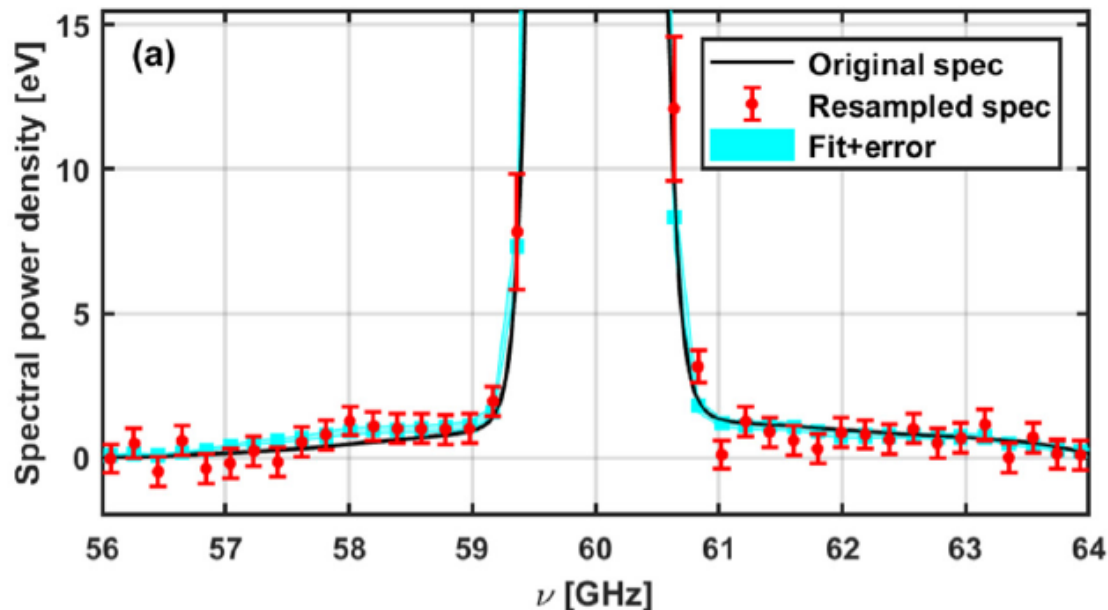
Contribution for each species depends on charge to mass ratio, thermal velocity and Larmor radius.

Description	Symbol	Unit	Value	σ_{prior}
<i>CTS system parameters</i>				
$\angle(\mathbf{k}^i, \mathbf{B}^{(0)})$	ϕ	°	88	3
$\angle(\mathbf{k}^i, \mathbf{k}^s)$	θ	°	140	3
Frequency of incident radiation	ν^i	GHz	60	
Mode of incident radiation			X	
Mode of scattered radiation			X	
Probe source power	P_{in}	W	10^6	
Spectral power density of receiver background	P_{b}	eV	100	
Time resolution	T	s	0.1	
Frequency resolution	W	MHz	5	
<i>Main plasma parameters</i>				
Magnetic field strength	B	T	5.3	10%
Electron density	n_e	10^{19} m^{-3}	11	5%
Electron temperature	T_e	keV	26.96	10%
Ion temperature	T_i	keV	23.49	10%
Rotation velocity	V_i	m s^{-1}	8.9×10^4	30%
<i>Plasma composition</i>				
Fuel ion ratio	R_i		1	10^6
Fast alpha particles	n_{α}/n_e		0.008	10%
Thermalized alpha particles	n_{He}/n_e		0.065	10%
Hydrogen	n_{H}/n_e		0.01	10%
Beryllium	n_{Be}/n_e		0.005	10%
Carbon	n_{C}/n_e		0.005	10%
Neon	n_{N}/n_e		2×10^{-4}	10%
Copper	n_{Cu}/n_e		10^{-4}	10%
Krypton	n_{Kr}/n_e		10^{-4}	10%
Tungsten	n_{W}/n_e		10^{-5}	10%

Stejner et al., Nucl. Fusion, 52, 023011 (2012)

ITER Coherent Thomson Scattering : fast ions

Synthetic ITER CTS spectrum



Fast ion contribution :

- Alpha particles
- Neutral Beam Injection particles

The spectrum is synthesized using ray-tracing.

3 different kinds of noise is incorporated.

Uncertainties on Bayesian priors

Fuel ion ratio $R_i = n_T / (n_D + n_T)$	24% ^c
Ion temperature T_i	10%
Ion drift velocity v_i	20%
Hydrogen density n_H	14% ^c
Thermal He densities n_{He3}, n_{He4}	20% ^d
Other impurity densities (Be, Ne, W)	10%
Electron density n_e	5%
Electron temperature T_e	1%

Rasmussen et al., Nucl. Fusion, 59, 096031 (2019)

Physics and Diagnostics in Tokamak plasmas

Thomson scattering response depends on $\vec{k}$ and ω

$k\lambda_D > 1$: Incoherent Thomson Scattering

The scattering signal spectrum reproduces electron velocity distribution

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{ev_k}(-\omega/k)$$

$k\lambda_D < 1$ $\omega_D \sim k v_{Ti}$: Coherent (or collective) Thomson Scattering

The signal spectrum is an image of the ion velocity distribution

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} \left[\left| 1 - \frac{\chi_e}{1 + \chi_e + \chi_i} \right|^2 f_{v_{ke}}\left(\frac{-\omega}{k}\right) + \sum_i \left| \frac{-\chi_e}{1 + \chi_e + \chi_i} \right|^2 Z_i^2 f_{v_{ki}}\left(\frac{-\omega}{k}\right) \right]$$

$k\lambda_D < 1$ $\omega_D \sim k c$: Collective Thomson Scattering

the signal spectrum might correspond plasma instabilities
with electron density fluctuations ($k, k c$)

$$S_{\vec{k}}(\omega) = \frac{1}{T N_s} |n_{eT}(\vec{k}, \omega)|^2$$

$$n_{\vec{k}}(t) = \iiint_{V_s} n_e(\vec{r}, t) e^{-i\vec{k} \cdot \vec{r}} d^3\vec{r}$$

Incoherent Thomson Scattering

Non perturbative measurements

Direct measurement

Spatially localized measurements with a good time resolution

Bibliography

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Chap. 10 : Diagnostics

I. H. Hutchinson, Principles of Plasma Diagnostics, 2nd ed, Cambridge Univ. Press (2002)

Chap. 7 : Scattering of electromagnetic radiation

D.H. Froula et al., Plasma Scattering of Electromagnetic Radiation, Academic Press, (1975)

J. D. Jackson, Classical Electrodynamics, J. Wiley (1975)

Chap. 9 : Simple Radiation Systems, Scattering, and Diffraction

Physical constants

$k_B = 1,38 \cdot 10^{-23} \text{ JK}^{-1}$: Boltzmann constant

$h = 6.62 \cdot 10^{-34} \text{ Js}$: Planck constant

$C = 2,99 \cdot 10^8 \text{ ms}^{-1}$: speed of light in vacuum

$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ Fm}^{-1}$: vacuum permittivity

$\mu_0 = 4\pi \cdot 10^{-7} \text{ Hm}^{-1}$: vacuum permeability

$q_e = 1,60 \cdot 10^{-19} \text{ C}$: elementary charge

$m_e = 9,11 \cdot 10^{-31} \text{ kg}$: electron mass

$r_e = \frac{1}{4\pi\epsilon_0} \frac{q_e^2}{m_e C^2} = 2.82 \cdot 10^{-15} \text{ m}$: electron classical radius

$N_A = 6,022 \cdot 10^{23} \text{ mol}^{-1}$: Avogadro constant

$m_u = 1,66 \cdot 10^{-27} \text{ kg}$: atomic mass unit

- Standard parameters

$T_0 = 273.15 \text{ K}$: standard air temperature (0°C)

$P_0 = 1,013 \cdot 10^5 \text{ Pa}$: standard air pressure

$n_0 = 2,69 \cdot 10^{25} \text{ m}^{-3}$: ideal gas molecular density at T_0 and P_0

- Units

$1 \text{ Torr} = \frac{1,013 \cdot 10^5}{760} \text{ Pa} = 133,3 \text{ Pa}$: pressure corresponding 1 mm of mercury

$1 \text{ eV} = \frac{1,6 \cdot 10^{-19}}{1,38 \cdot 10^{-23}} \text{ K} = 1,16 \cdot 10^4 \text{ K}$